MEP 579 Applications of Industrial Pipe Lines – Total Summary of Piping Networks

Steady Incompressible Flow in Pipe lines or Pressure Conduits

In this chapter some of the aspects of steady flow in pressure conduits are discussed. The discussion is limited to *incompressible fluids*, that is, to those for which $\rho \approx$ constant. This includes all liquids. In this chapter isothermal conditions are assumed so as to eliminate thermodynamic effects, some of which are discussed in Chap. 9. Gases flowing with very small pressure changes may be considered *incompressible*, for then $\rho \approx$ constant.

8.1 LAMINAR AND TURBULENT FLOW

If the head loss in a given length of uniform pipe is measured at different values of the velocity, it will be found that, as long as the velocity is low enough to secure laminar flow, the head loss, due to friction, will be directly proportional to the velocity, as shown in Fig. 8.1. But with increasing velocity, at some point B, where visual observation in a transparent tube would show that the flow changes from laminar to turbulent, there will be an abrupt increase in the rate at which the head loss varies. If the logarithms of these two variables are plotted on linear scales or if the values are plotted directly on log-log paper, it will be found that, after a certain transition region has been passed, lines will be obtained with slopes ranging from about 1.75 to 2.00.

It is thus seen that for laminar flow the drop in energy due to friction varies as V, while for turbulent flow the friction varies as V^n, where n ranges from about 1.75 to 2. The lower value of 1.75 for turbulent flow is found for pipes with very smooth walls; as the wall roughness increases, the value of n increases up to its maximum value of 2.

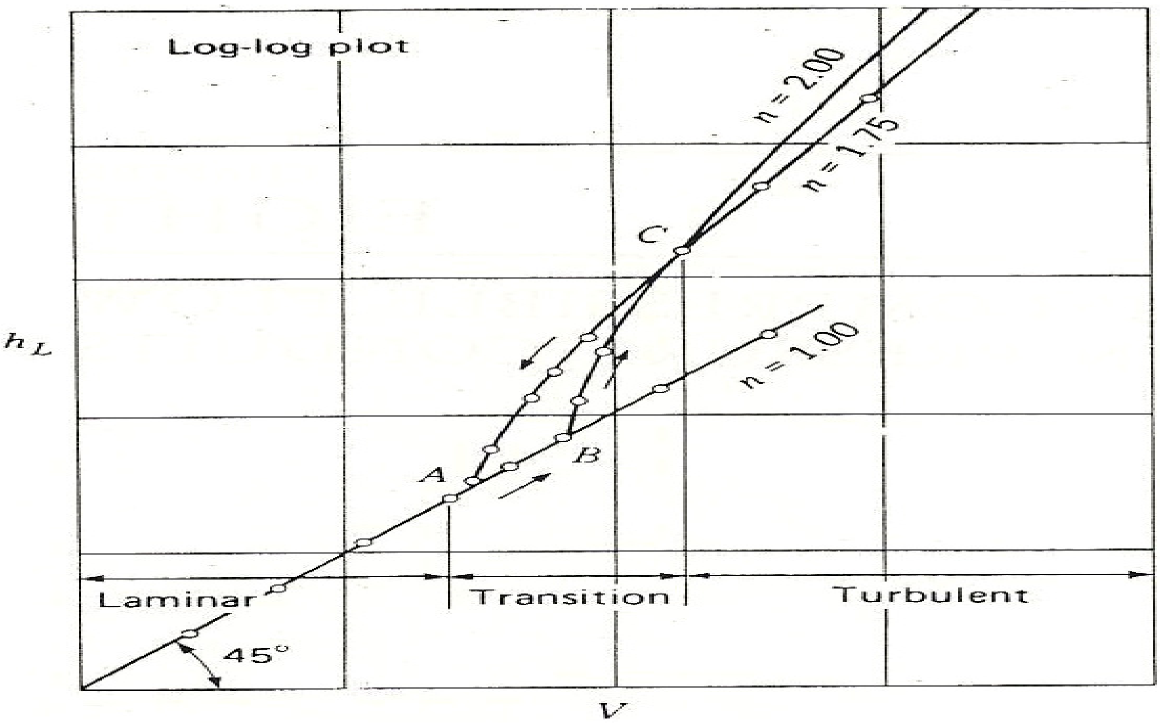


Figure 8.1 Log-log plot for flow in a uniform pipe. ($n = 2.00$, rough-wall pipe; $n = 1.75$ smooth-wall pipe.)

The points in Fig. 8.1 were plotted directly from Osborne Reynolds' measurements and show decided curves in the transition zone where values of n are even greater than 2. If the velocity is gradually reduced from a high value, the line BC will not be retraced. Instead, the points lie along curve CA. Point B is known as the *higher critical point*, and A as the *lower critical point*.

However, velocity is not the only factor that determines whether the flow is laminar or turbulent. The criterion is Reynolds number, which has been discussed in Sec. 7.4. For a circular pipe the significant linear dimension L is usually taken as the diameter D, and thus

$$\mathbf{R} = \frac{DV\rho}{\mu} = \frac{DV}{\nu} \tag{8.1}$$

where any consistent system of units may be used, since $\mathbf{R}$ is a dimensionless number.[1]

[1] It is sometimes convenient to use a "hybrid" set of units and compensate with a correction factor. Thus by substituting $V = Q/A$ and $V = G/\gamma A$ into Eq. (8.1), we get

$$\mathbf{R} = 1.27Q/\nu D = 1.27G/\mu g D,$$

where Q and G are defined in the Notation in the front of the book. The last form is especially convenient in the case of gases; it shows that in a pipe of uniform diameter the Reynolds number is constant along the pipe, even for a compressible fluid where the density and velocity vary, if there is no appreciable variation in temperature to alter the viscosity of the gas.

8.2 CRITICAL REYNOLDS NUMBER

The upper critical Reynolds number, corresponding to point B of Fig. 8.1, is really indeterminate and depends upon the care taken to prevent any initial disturbance from affecting the flow. Its value is normally about 4,000, but laminar flow in circular pipes has been maintained up to values of $\mathbf{R}$ as high as 50,000. However, in such cases this type of flow is inherently unstable, and the least disturbance will transform it instantly into turbulent flow. On the other hand, it is practically impossible for turbulent flow in a straight pipe to persist at values of $\mathbf{R}$ much below 2,000, because any turbulence that is set up will be damped out by viscous friction. This lower value is thus much more definite than the higher one and is really the dividing point between the two types of flow. Hence this lower value will be defined as the *true critical Reynolds number*. However, this lower critical value is subject to slight variations. Its value will be higher in a converging pipe and lower in a diverging pipe than in a straight pipe. Also, its value will be less for flow in a curved pipe than in a straight one, and even for a straight uniform pipe its value may be as low as 1,000, where there is an excessive degree of roughness. However, for normal cases of flow in straight pipes of uniform diameter and usual roughness, the critical value may be taken as $\mathbf{R} = 2{,}000$.

For water at 20°C the kinematic viscosity is 1.00×10^{-6} m²/s, and for this case the critical Reynolds number is obtained when

$$DV_{\text{crit}} = \mathbf{R}\nu = 2{,}000 \times 10^{-6}\ \text{m}^2/\text{s} = 0.002\ \text{m}^2/\text{s}$$

Thus, for a pipe 25 mm in diameter, $\quad V_{\text{crit}} = 0.002/0.025 = 0.08$ m/s

Or if the velocity were 0.8 m/s the diameter would be only 2.5 mm. Velocities or pipe diameters as small as these are not often encountered with water flowing in practical engineering, though they may be found in certain laboratory instruments. Hence, for such fluids as water and air, practically all cases of engineering importance are in the turbulent-flow region. But if the fluid is a viscous oil, laminar flow is often encountered.

Illustrative Example 8.1 An oil ($s = 0.85$, $\nu = 1.8 \times 10^{-5}$ m²/s) flows in a 10-cm-diameter pipe at 0.50 L/s. Is the flow laminar or turbulent?

$$V = \frac{Q}{A} = \frac{500 \text{ cm}^3/\text{s}}{\pi(10)^2 \text{ cm}^2/4} = 6.37 \text{ cm/s} = 0.0637 \text{ m/s}$$

$$\mathbf{R} = \frac{DV}{\nu} = \frac{0.10 \text{ m}(0.0637 \text{ m/s})}{1.8 \times 10^{-5} \text{ m}^2/\text{s}} = 354$$ Since $\mathbf{R} < 2{,}000$, the flow is laminar.

8.3 HYDRAULIC RADIUS

For conduits having noncircular cross sections, some value other than the diameter must be used for the linear dimension in the Reynolds number. Such a characteristic is the *hydraulic radius*, defined as

$$R_h = \frac{A}{P} \tag{8.2}$$

where A is the cross-sectional area of the flowing fluid, and P is the *wetted perimeter*, that portion of the perimeter of the cross section where there is contact between fluid and solid boundary. For a circular pipe flowing full, $R_h = \pi r^2/2\pi r = r/2$, or $D/4$. Thus R_h is not the radius of the pipe, and hence the term "radius" is misleading. If a circular pipe is exactly half full, both the area and the wetted perimeter are half the preceding values; so R_h is $r/2$, the same as if it were full. But if the depth of flow in a circular pipe is 0.8 times the diameter, for example, $A = 0.674D^2$ and $P = 2.21D$, then $R_h = 0.304D$, or $0.608r$.

The hydraulic radius is a convenient means for expressing the shape as well as the size of a conduit, since for the same cross-sectional area the value of R_h will vary with the shape.

In evaluating the Reynolds number for flow in a noncircular conduit (Sec. 8.13) it is customary to substitute $4R_h$ for D in Eq. (8.1).

8.4 GENERAL EQUATION FOR CONDUIT FRICTION

The following discussion applies to either laminar or turbulent flow and to any shape of cross section.

Consider steady flow in a conduit[1] of uniform cross section A (Fig. 8.2). The pressures at sections 1 and 2 are p_1 and p_2, respectively. The distance between

[1] This conduit can have any shape of cross section; it need not be circular.

sections is L. For equilibrium in steady flow, the summation of forces acting on any fluid element must be equal to zero (i.e., $\sum F = ma = 0$). Thus, in the direction of flow,

$$p_1 A - p_2 A - \gamma L A \sin\alpha - \bar{\tau}_0(PL) = 0 \tag{8.3}$$

where $\bar{\tau}_0$, the *average shear stress* (average shear force per unit area) at the conduit wall, is defined by

$$\bar{\tau}_0 = \frac{\int_0^P \tau_0 \, dP}{P} \tag{8.4}$$

in which τ_0 is the local shear stress[1] acting over a small incremental portion dP of the wetted perimeter.

[1] The local shear stress varies from point to point around the perimeter of all conduits (irrespective of whether the wall is smooth or rough) except for the case of a circular pipe flowing full where the shear stress at the wall is the same at all points of the perimeter.

[2] Here we are using the *FLT* system, while in Chap. 7 the *MLT* system was used. It makes no difference which system is used since the results are the same.

Noting that $\sin\alpha = (z_2 - z_1)/L$ and dividing each term in Eq. (8.3) by γA gives

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - z_2 + z_1 = \bar{\tau}_0 \frac{PL}{\gamma A} \tag{8.5}$$

From the left-hand sketch of Fig. 8.2 it can be seen that

$$h_L = (z_1 + p_1/\gamma) - (z_2 + p_2/\gamma)$$

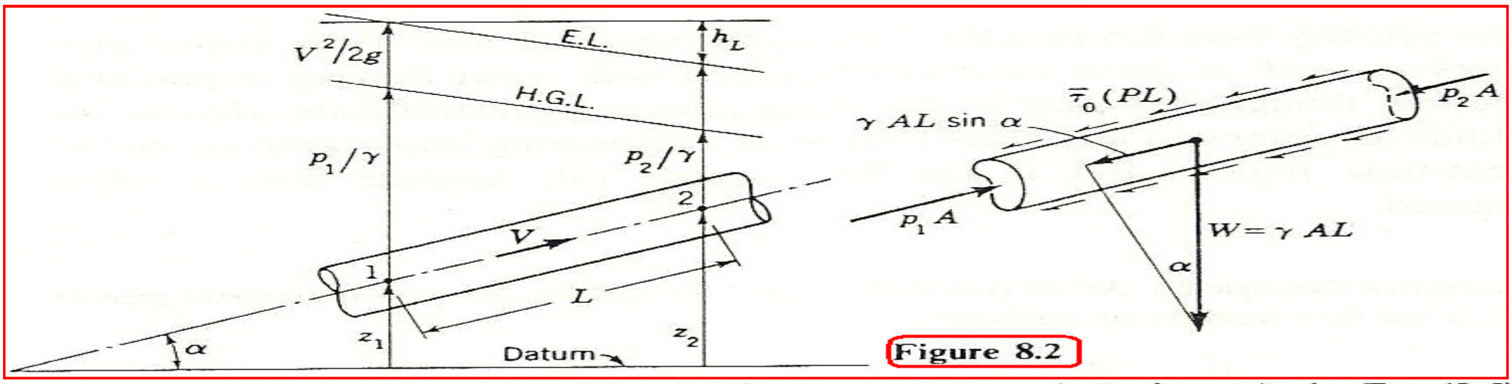


Figure 8.2

Substituting h_L for the right side of this expression and R_h for A/P in Eq. (8.5) we get,

$$h_L = \bar{\tau}_0 \frac{L}{R_h \gamma} \tag{8.6}$$

This equation is applicable to any shape of uniform cross section regardless of whether the flow is laminar or turbulent.

For a smooth-walled conduit, where wall roughness (discussed in Sec. 8.9) may be neglected, it might be assumed that the average fluid shear stress $\bar{\tau}_0$ at the wall is some function of ρ, μ, V and some characteristic linear dimension, which will here be taken as the hydraulic radius R_h. Thus

$$\bar{\tau}_0 = K R_h^a \rho^b \mu^c V^n \tag{8.7}$$

where K is a dimensionless number. Substituting in Eq. (8.7) dimensional values of F, L, and T for force, length, and time, we get[2]

$$FL^{-2} = KL^a(FL^{-4}T^2)^b(FL^{-2}T)^c(LT^{-1})^n$$

As the dimensions on the two sides of the equation must be the same,

For F: $1 = b + c$ For L: $-2 = a - 4b - 2c + n$ For T: $0 = 2b + c + n$

The solution of these three simultaneous expressions in terms of n is $a = n - 2$, $b = n - 1$, $c = 2 - n$.

Inserting these values of the exponents in Eq. (8.7), the result is

$$\bar{\tau}_0 = K R_h^{n-2} \rho^{n-1} \mu^{2-n} V^n \tag{8.8}$$

This may be rearranged as

$$\bar{\tau}_0 = K\left(\frac{R_h V \rho}{\mu}\right)^{n-2} \rho V^2 = 2K\mathbf{R}^{n-2} \rho \frac{V^2}{2} \tag{8.9}$$

for $R_h V\rho/\mu$ is a Reynolds number with R_h as the characteristic length.

Grouping the dimensionless terms on the right side of Eq. (8.9) into a single term C_f, we get

$$C_f = 2K\mathbf{R}^{n-2} \tag{8.10}$$

Hence
$$\bar{\tau}_0 = C_f \rho \frac{V^2}{2} \tag{8.11}$$

Inserting this value of $\bar{\tau}_0$ in Eq. (8.6) and noting that $\gamma = \rho g$,

$$h_L = C_f \frac{L}{R_h} \frac{V^2}{2g} \tag{8.12}$$

which may be applied to any shape of smooth-walled cross section. Later it will be shown (Sec. 8.11) that this equation also applies to rough-walled conduits.

8.5 PIPES OF CIRCULAR CROSS SECTION

In Sec. 8.3 it is shown that for a circular pipe flowing full $R_h = D/4$. Substituting this value in Eq. (8.12), the result (for both smooth-walled and rough-walled conduits) is

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \tag{8.13}$$

where
$$f = 4C_f = 8K\mathbf{R}^{n-2} \tag{8.14}$$

Equation (8.13) is known as the *pipe-friction equation*, and is also commonly referred to as the Darcy-Weisbach equation.[1] Like the coefficient C_f, the friction factor f is dimensionless and is also some function of Reynolds number. Much

[1] In a slightly different form where D is replaced by the hydraulic radius R_h, Eq. (8.13) is known as the Fanning equation which is widely used by chemical engineers.

research has been directed toward determining the way in which f varies with **R** and also with pipe roughness. The pipe-friction equation expresses the fact that the head lost in friction in a given pipe can be expressed in terms of the velocity head. The equation is dimensionally homogeneous and may be used with any consistent system of units.

Dimensional analysis gives us the proper form for an equation but does not yield a numerical result since it is not concerned with abstract numerical factors. Hence it shows in Eq. (8.8) that whatever the value of the exponent of V, the exponents of all the other quantities involved are then determined. It also shows that Eq. (8.13) is a rational expression for pipe friction. But the numerical values of such quantities as K, n, and f must be determined by experiment or other means. For a circular pipe flowing full, Eq. (8.6) may be written as

$$h_L = \bar{\tau}_0 \frac{L}{R_h \gamma} = \frac{\tau_0 2L}{r_0 \gamma} \tag{8.15}$$

where the shear stress at the wall, $\tau_0 = \bar{\tau}_0$, because of symmetry, and $R_h = r_0/2$ where r_0 is the radius of the pipe.

Following a development similar to that of Eqs. (8.3) to (8.6) and noting for a circle that $A = \pi r^2$ and $P = 2\pi r$, it can be shown that for a cylindrical body of fluid concentric to the pipe, $h_L = \tau 2L/r\gamma$ where τ is the shear stress in the fluid at radius r. Relating this to Eq. (8.15) it follows that the shear stress in the flow in a circular pipe at any radius r is

$$\tau = \tau_0 \frac{r}{r_0} \tag{8.16}$$

or the shear stress is zero at the center of the pipe and increases linearly with the radius to a maximum value τ_0 at the wall as in Fig. 8.3. This is true regardless of whether the flow is laminar or turbulent.

From Eqs. (8.6) and (8.13) and substituting $R_h = D/4$ for a circular pipe, we obtain

$$\tau_0 = \frac{f}{4} \rho \frac{V^2}{2} = \frac{f}{4} \gamma \frac{V^2}{2g} \tag{8.17}$$

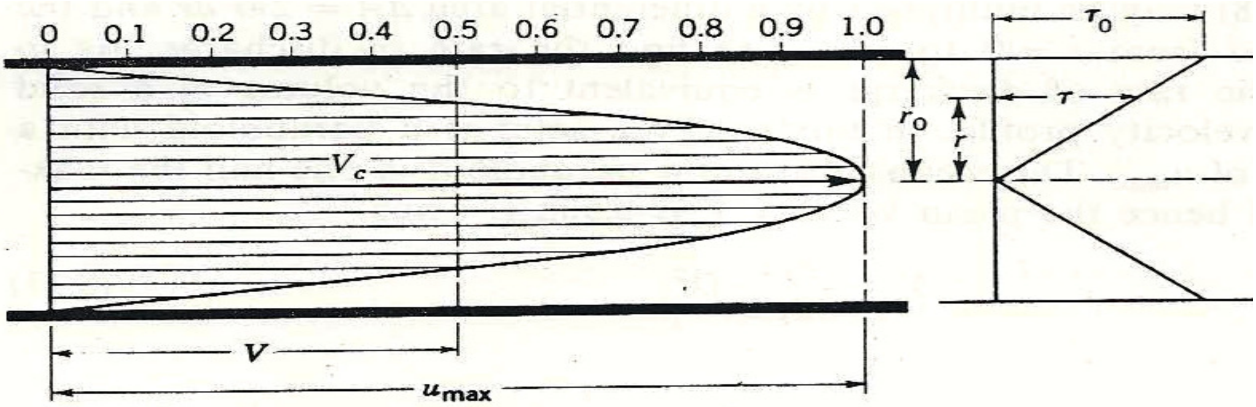


Figure 8.3 Velocity profile in laminar flow and distribution of shear stress.

With this equation, τ_0 for flow in a circular pipe may be computed for any experimentally determined value of f.

8.6 LAMINAR FLOW IN CIRCULAR PIPES

In Sec. 1.11 it was noted that for laminar flow $\tau = \mu\, du/dy$, where u is the value of the velocity at a distance y from the boundary. As $y = r_0 - r$, it is also seen that $\tau = -\mu\, du/dr$; in other words, the minus sign indicates that u decreases as r increases. The coefficient of viscosity μ is a constant for any particular fluid at a constant temperature, and therefore if the shear varies from zero at the center of the pipe to a maximum at the wall, it follows that the velocity profile must have a zero slope at the center and have a continuously steeper velocity gradient as the wall is approached.

To determine the velocity profile for laminar flow in a circular pipe the expression $\tau = \mu\, du/dy$ is substituted into the expression $h_L = \tau 2L/r\gamma$.

Thus
$$h_L = \frac{\tau 2L}{r\gamma} = \mu \frac{du}{dy}\frac{2L}{r\gamma} = -\mu \frac{du}{dr}\frac{2L}{r\gamma}$$

From this
$$du = -\frac{h_L \gamma}{2\mu L} r\, dr$$

Integrating and determining the constant of integration from the fact that $u = u_{max}$ when $r = 0$, we obtain

$$u = u_{max} - \frac{h_L \gamma}{4\mu L} r^2 = u_{max} - kr^2 \tag{8.18}$$

From this equation it is seen that the velocity profile is a parabola, as shown in Fig. 8.3. Note that $k = h_r \gamma / 4\mu L$.

Substituting the boundary condition that $u = 0$ when $r = r_0$ into the second expression of Eq. (8.18) and noting that $u_{max} = V_c$, the centerline velocity, we find $k = V_c / r_0^2$. Thus Eq. (8.18) can be expressed as

$$u = V_c - (V_c/r_0^2) r^2 = V_c(1 - r^2/r_0^2) \tag{8.19}$$

Combining Eqs. (8.18) and (8.19) we get an expression for V_c as follows

$$V_c = u_{max} = \frac{h_L \gamma}{4\mu L} r_0^2 = \frac{h_L \gamma}{16\mu L} D^2 \tag{8.20}$$

Equation (8.18) may be multiplied by a differential area $dA = 2\pi r\, dr$ and the product integrated from $r = 0$ to $r = r_0$ to find the rate of discharge. As in previous cases, the rate of discharge is equivalent to the volume of a solid bounded by the velocity profile. In this case the solid is a paraboloid with a maximum height of u_{max}. The mean height of a paraboloid is one-half the maximum height, and hence the mean velocity V is $0.5u_{max}$. Thus

$$V = \frac{h_L \gamma}{32\mu L} D^2 \tag{8.21}$$

From this last equation, noting that $\gamma = g\rho$ and $\mu/\rho = \nu$, the loss of head in friction is given by

$$h_L = 32 \frac{\mu}{\gamma} \frac{L}{D^2} V = 32\nu \frac{L}{gD^2} V \tag{8.22}$$

which is the Hagen-Poiseuille law for laminar flow in tubes. Hagen, a German engineer, experimented with water flowing through small brass tubes and published his results in 1839. Poiseuille, a French scientist, experimented with water flowing through capillary tubes in order to determine the laws of flow of blood through the veins of the body and published his studies in 1840.

From Eq. (8.22) it is seen that in laminar flow the loss of head is proportional to the first power of the velocity. This is verified by experiment, as shown in Fig. 8.1. The striking feature of this equation is that it involves no empirical coefficients or experimental factors of any kind, except for the physical properties of the fluid such as viscosity and density (or specific weight). From this it would appear that in laminar flow the friction is independent of the roughness of the pipe wall. That this is true is also borne out by experiment.

Dimensional analysis shows that the friction loss may also be expressed by Eq. (8.13). Equating (8.13) and (8.22) and solving for the friction factor f, we obtain for laminar flow under pressure in a circular pipe,

$$f = \frac{64\nu}{DV} = \frac{64}{\mathbf{R}} \tag{8.23}$$

Hence, if $\mathbf{R}$ is less than 2,000, we may use Eq. (8.22) to find pipe friction head loss or we may use Eq. (8.13) with the value of f as given by Eq. (8.23).

8.7 ENTRANCE CONDITIONS IN LAMINAR FLOW

In the case of a pipe leading from a reservoir, if the entrance is rounded so as to avoid any initial disturbance of the entering stream, all particles will start to flow with the same velocity, except for a very thin film in contact with the wall. Particles next to the wall have a zero velocity, but the velocity gradient is here extremely steep, and with this slight exception, the velocity is uniform across the diameter, as shown in Fig. 8.4. As the fluid progresses along the pipe, the streamlines in the vicinity of the wall are slowed down by friction emanating from the wall, but as Q is constant for successive sections, the velocity in the center must be accelerated, until the final velocity profile is a parabola, as shown in Fig. 8.3. Theoretically, an infinite distance is required for this, but it has been established both by theory and by observation that the maximum velocity in the center of the pipe will reach 99 per cent of its ultimate value in the distance $L' = 0.058\mathbf{R}D$.[1] Thus, for the critical value $\mathbf{R} = 2{,}000$, the distance L' of Fig. 8.4 equals 116 pipe

[1] H. L. Langhaar, Steady Flow in the Transition Length of a Straight Tube, *J. Appl. Mech.*, vol. 10, p. 55, 1942.

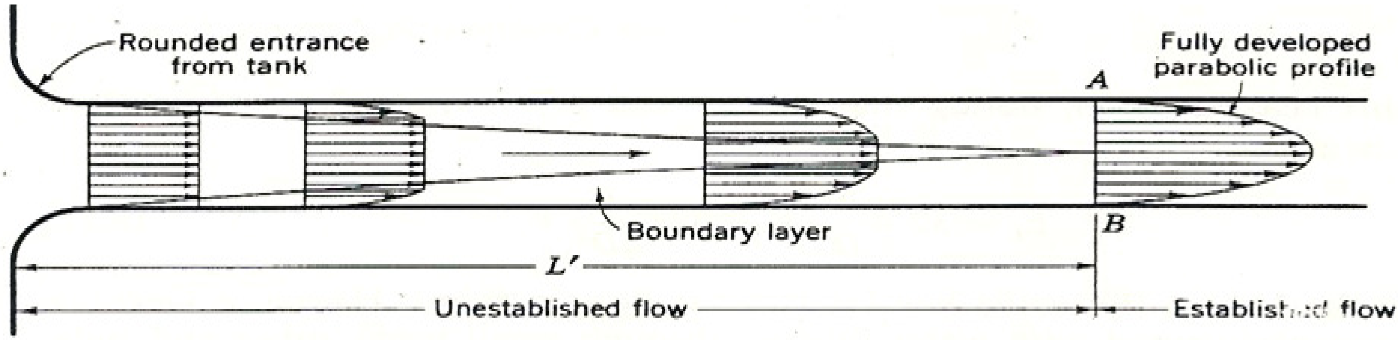


Figure 8.4 Velocity profiles along a pipe in laminar flow.

diameters. In other cases of laminar flow with Reynolds numbers less than 2,000, the distance L' will be correspondingly less in accordance with the expression $L' = 0.058\mathbf{R}D$.

In the entry region of length L' the flow is *unestablished*; that is, the velocity profile is changing. In this region the flow can be visualized as consisting of a central core in which there are no frictional effects and an annular zone extending from the core outward to the pipe wall. This outer zone increases in thickness as it moves along the wall and is known as the *boundary layer*. Viscosity in the boundary layer acts to transmit the effect of boundary shear inwardly into the flow. At section AB the boundary layer has grown until it occupies the entire section of the pipe. At this point, for laminar flow, the velocity profile is a perfect parabola. Beyond section AB the velocity profile does not change, and the flow is known as *established flow*.

As shown in Prob. 4.1 for a circular pipe, the kinetic energy of a stream with a parabolic velocity profile is $2V^2/2g$, where V is the mean velocity. At the entrance to the pipe the velocity is uniformly V across the diameter, except for an extremely thin layer next to the wall. Thus, at the entrance to the pipe, the kinetic energy per unit weight is practically $V^2/2g$. Hence, in the distance L', there is a continuous increase in kinetic energy accompanied by a corresponding decrease in pressure head. Therefore, at a distance L' from the entrance, the piezometric head is less than the static value in the reservoir by $2V^2/2g$ plus the friction loss in this distance.

Laminar flow has been dealt with rather fully, not merely because it is of importance in problems involving fluids of very high viscosity, but especially because it permits a simple and accurate rational analysis. The general approach used here is of some assistance in the study of turbulent flow, where conditions are so complex that rigid mathematical treatment is impossible.

Illustrative Example 8.2 For the case of Illustrative Example 8.1 find the centerline velocity, the velocity at $r = 2$ cm, the friction factor, the shear stress at the pipe wall, and the head loss per meter of pipe length.

Since the flow is laminar, $V_c = 2V = 12.7$ cm/s $\quad u = u_{max} - kr^2 \quad u_{max} = V_c = 12.7$ cm/s

When $r = r_0 = 5$ cm, $u = 0$, hence $0 = 12.7 - k(5)^2$

$k = 0.51/(\text{cm} \cdot \text{s}) \quad u_{2\,\text{cm}} = 12.7 - 0.51(2)^2 = 10.7$ cm/s

$$f = \frac{64}{\mathbf{R}} = \frac{64}{354} = 0.18 \qquad \tau_0 = \frac{f}{4}\rho\frac{V^2}{2} = \frac{0.18}{4}(0.85\ \text{g/cm}^3)\frac{(6.37\ \text{cm/s})^2}{2}$$

$$\tau_0 = 0.77\frac{g}{(\text{cm}\cdot\text{s}^2)}\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\frac{100\ \text{cm}}{\text{m}} = 0.077\ \text{N/m}^2$$

$$h_L/L = f\frac{1}{D}\frac{V^2}{2g} = 0.18\frac{1}{0.10\ \text{m}}\frac{(0.0637\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 0.00037\ \text{m/m}$$

8.8 TURBULENT FLOW

In Sec. 3.1 it was explained that in laminar flow the fluid particles move in straight lines while in turbulent flow they follow random paths. Consider the case of laminar flow as shown in Figs. 8.5*a* and 8.5*b* where the velocity u increases with y. Even though the fluid particles are moving horizontally to the right, because of molecular motion, molecules will cross line *ab* and will thereby transport momentum. On the average, the velocities of the molecules in the slower moving fluid below the line will be less than those of the faster moving fluid above; the result is that the molecules which cross from below tend to slow down the faster moving fluid. Likewise, the molecules which cross the line *ab* from above tend to speed up the slower moving fluid below. The result is the production of a shear stress along the surface whose trace is *ab*, the value of which is given in Sec. 1.11 as $\tau = \mu \, du/dy$. This equation is applicable to laminar flow only.

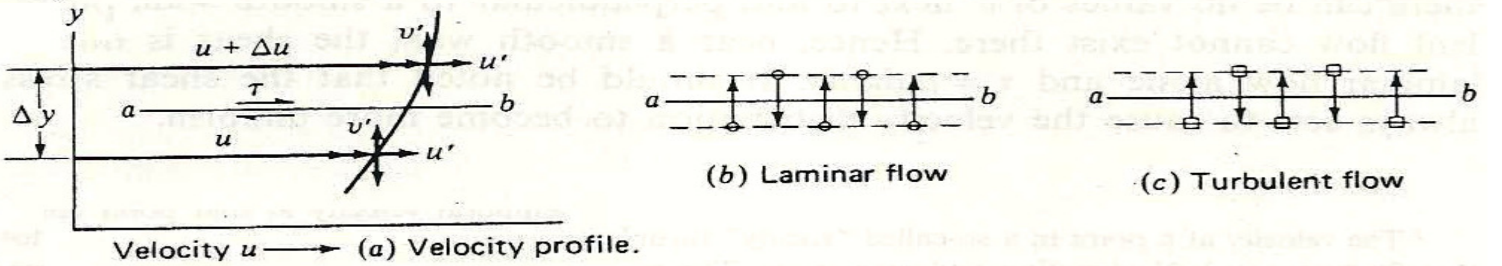


Figure 8.5 (*a*) Velocity profile. (*b*) Laminar flow (transfer of molecules across *ab*). (c) Turbulent flow (transfer of finite fluid masses across *ab*).

Let us examine some of the characteristics of turbulent flow to see how it differs from laminar flow. In turbulent flow the velocity at a point in the flow field fluctuates in both magnitude and direction.[1] As a consequence a multitude of small eddies are created by the viscous shear between adjacent particles. These eddies grow in size and then disappear as their particles merge into adjacent eddies. Thus there is a continuous mixing of particles, with a consequent transfer of momentum.

First Expression

In the modern conception of turbulent flow, a mechanism similar to that described in the foregoing for laminar flow is assumed. However, the molecules are replaced by minute but finite masses (Fig. 8.5*c*). Hence, by analogy, the shear stress along the plane through *ab* in Fig. 8.5 may be defined in the case of turbulent flow as

$$\text{Turbulent shear stress} = \eta \frac{du}{dy} \tag{8.24}$$

But unlike μ, the *eddy-viscosity* η is not a constant for a given fluid at a given temperature, but depends upon the turbulence of the flow. It may be viewed as a coefficient of momentum transfer, expressing the transfer of momentum from points where the velocity is low to points where it is higher, and vice versa. Its magnitude may range from zero to many thousand times the value of μ. However, its numerical value is of less interest than its physical concept. In dealing with turbulent flow it is sometimes convenient to use *kinematic eddy viscosity* $\varepsilon = \eta/\rho$ which is a property of the flow alone, analogous to kinematic viscosity.

In general, the total shear stress in turbulent flow is the sum of the laminar shear stress plus the turbulent shear stress, i.e.,

$$\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy} = \rho(\nu + \varepsilon)\frac{du}{dy} \tag{8.25}$$

In turbulent flow the second term of this equation is usually many times larger than the first term.

In turbulent flow the local axial velocity has been shown, in Sec. 3.4 (see Fig. 3.6), to have fluctuations of plus and minus u', and there are also fluctuations of plus and minus v' and w' normal to u as shown in Fig. 8.6*b*. As it is obvious that there can be no values of v' next to and perpendicular to a smooth wall, turbulent flow cannot exist there. Hence, near a smooth wall, the shear is due to laminar flow alone and $\tau = \mu \, du/dy$. It should be noted that the shear stress always acts to cause the velocity distribution to become more uniform.

[1] The velocity at a point in a so-called "steady" turbulent flow can be best visualized as a vector that fluctuates in both direction and magnitude. The mean temporal velocity at that point corresponds to the "average" of those vectors.

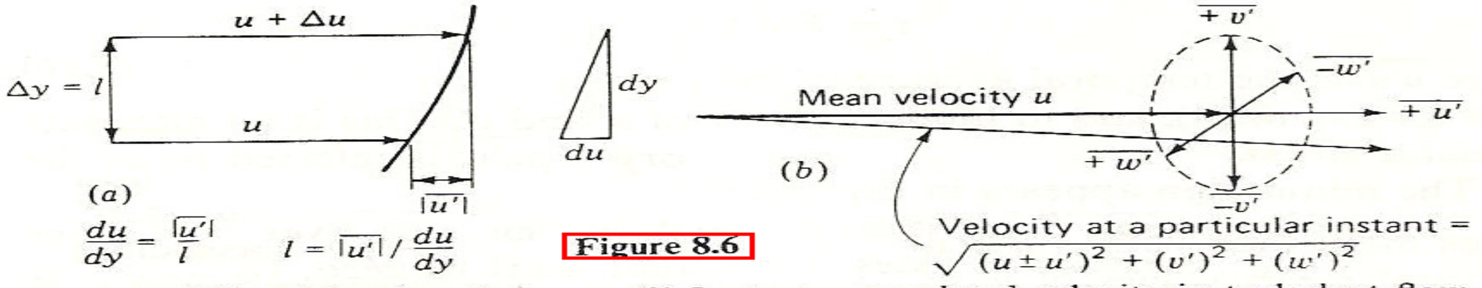


Figure 8.6

(*a*) Prandtl's mixing length *l*. (*b*) Instantaneous local velocity in turbulent flow.

At some distance from the wall, such as $0.2r$, the value of du/dy becomes small in turbulent flow, and hence the viscous shear becomes negligible in comparison with the turbulent shear. The latter can be large, even though du/dy is small, because of the possibility of η being very large. This is due to the great turbulence that may exist at an appreciable distance from the wall. But at the center of the pipe, where du/dy is zero, there can be no shear at all. Hence, in turbulent flow as well as in laminar flow, the shear stress is a maximum at the wall and decreases linearly to zero at the axis, as shown in Fig. 8.3 and proved in Sec. 8.5.

Second Expression

Another expression for turbulent shear stress may be obtained which is different from that in Eq. (8.24). Thus in Fig. 8.5*a*, if a mass *m* of fluid below *ab*, where the temporal mean axial velocity is *u*, moves upward into a zone where the temporal mean axial velocity is $u + \Delta u$, its initial momentum in the axial direction must be increased by $m\,\Delta u$. Conversely, a mass *m* which moves from the upper zone to the lower will have its axial momentum decreased by $m\,\Delta u$. Hence this transfer of momentum back and forth across *ab* will produce a shear in the plane through *ab* proportional to Δu. This shear is possible only because of the velocity profile shown. If the latter were vertical, Δu would be zero and there could be no shear.

If the distance Δy in Fig. 8.5*a* is so chosen that the average value of $+u'$ in the upper zone over a time period long enough to include many velocity fluctuations is equal to Δu, i.e., $\Delta u = |\overline{u'}|$, the two streams will be separated by what is known as the *Prandtl mixing length l*, which will be referred to later. Consider, over a short time interval, a mass moving upward from below *ab* with a velocity v'; it will transport into the upper zone, where the velocity is $u + u'$, a momentum per unit time which is on the average equal to $\rho(v'\,dA)(u)$. The slower moving mass from below *ab* will tend to retard the flow above *ab*; this creates a shear force along the plane of *ab*. This force can be found by applying the momentum principle [Eq. (6.6)], $F = \tau\,dA = \rho Q(\Delta V) = \rho(v'\,dA)(u + u' - u) = \rho u'v'\,dA$. Thus, over a time period of sufficient length to permit a large number of velocity fluctuations, the shear stress given by

$$\tau = F/dA = -\rho\overline{u'v'} \tag{8.26}$$

where $\overline{u'v'}$ is the temporal average of the product of u' and v'. This is an alternate form for Eq. (8.24), and in modern turbulence theory $-\rho\overline{u'v'}$ is referred to as the *Reynolds stress.*

The minus sign appears in Eq. (8.26) because the product $\overline{u'v'}$ on the average is negative. By inspecting Fig. 8.5*a* it can be seen that $+v'$ is associated with $-u'$ values more than with $+u'$ values. The opposite is true for $-v'$. Even though the temporal mean values of u' and v' are individually equal to zero, the temporal mean value of their product is not zero. This is so because combinations of $+v'$ and $-u'$ and of $-v'$ and $+u'$ predominate over combinations of $+v'$ and $+u'$ and $-v'$ and $-u'$ respectively.

Prandtl reasoned that in any turbulent flow $|\overline{u'}|$ and $|\overline{v'}|$ must be proportional to each other and of the same order of magnitude. He also introduced the concept of mixing length *l*, which is defined as the distance one must move transversely to the direction of flow such that $\Delta u = |\overline{u'}|$. From Fig. 8.6*a* it can be seen that $\Delta u = l\,du/dy$ and hence $|\overline{u'}| = l\,du/dy$. If $|\overline{u'}| \propto |\overline{v'}|$ and if one permits *l* to account for the constant of proportionality, Prandtl[1] has shown that $-\overline{u'v'}$ varies as $l^2(du/dy)^2$. Thus

$$\tau = -\rho\overline{u'v'} = \rho l^2\left(\frac{du}{dy}\right)^2 \tag{8.27}$$

This equation expresses terms that can be measured. Thus in any experiment where the pipe friction is determined, τ_0 can be computed by Eq. (8.5), and τ at any radius is then found by Eq. (8.14). A traverse of the velocity across a pipe diameter will give *u* at any radius, and the velocity profile will give du/dy at any radius. Thus Eq. (8.27) enables the Prandtl mixing length *l* to be found as a function of the pipe radius. The purpose of all of this is to enable us to develop theoretical equations for the velocity profile in turbulent flow, and from this in turn to develop theoretical equations for *f*, the friction coefficient.

8.9 VISCOUS SUBLAYER IN TURBULENT FLOW

In Fig. 8.4 it is shown that, for laminar flow, if the fluid enters with no initial disturbance, the velocity is uniform across the diameter except for an exceedingly thin film at the wall, inasmuch as the velocity next to any wall is zero. But as flow proceeds down the pipe, the velocity profile changes because of the growth of a laminar boundary layer which continues until the boundary layers from opposite sides meet at the pipe axis and then there is fully developed laminar flow.

If the Reynolds number is above the critical value, so that the flow is turbulent, the initial condition is much like that in Fig. 8.4. But as the laminar boundary layer increases in thickness, a point is soon reached where a transition occurs and the boundary layer becomes turbulent. This turbulent boundary layer generally increases in thickness much more rapidly, and soon the two from opposite sides meet at the pipe axis, and there is then fully developed turbulent flow.

This initial laminar boundary layer may be given a Reynolds number such as $\mathbf{R}_x = Ux/\nu$, where U is the uniform velocity and x is the distance measured from the initial point. When x has such a value that this $\mathbf{R}_x$ is about 500,000, the transition occurs to the turbulent boundary layer. Fully developed turbulent flow will be found at about 50 pipe diameters from the pipe entrance for a smooth pipe with no special disturbance at entrance; otherwise the turbulent boundary layers from the two sides will meet within a shorter distance. It is this fully developed turbulent flow that we shall consider in all that follows.

There can be no turbulence next to a smooth wall since it is impossible for v' to have any value at a solid boundary. Therefore immediately adjacent to a smooth wall there will be a laminar or *viscous* sublayer, as shown in Fig. 8.7, within which the shear is due to viscosity alone. This viscous sublayer is extremely thin, usually only a few hundredths of a millimeter, but its effect is great because of the very steep velocity gradient within it and because $\tau = \mu\, du/dy$ in that region. At a distance from the wall the viscous effect becomes negligible, but the turbulent shear is then large. Between the two there must be a transition zone where both types of shear are significant. It is evident that there can be no sharp lines of demarcation separating these three zones, inasmuch as one must merge gradually into the other.

By plotting a velocity profile from the wall on the assumption that the flow is entirely laminar (Sec. 8.6) and plotting another velocity profile on the assumption that the flow is entirely turbulent (Sec. 8.10), the two will intersect, as shown in Fig. 8.8. It is obvious that there can be no abrupt change in profile at this point

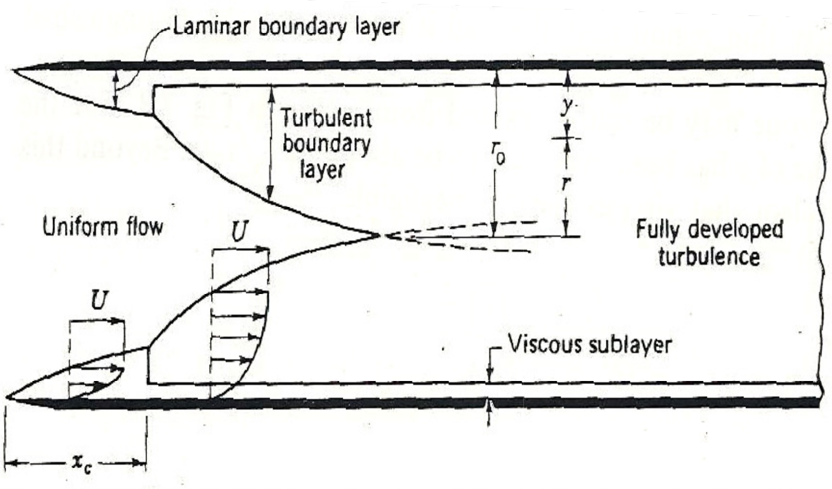


Figure 8.7 Development of boundary layer in a pipe (scales much distorted).

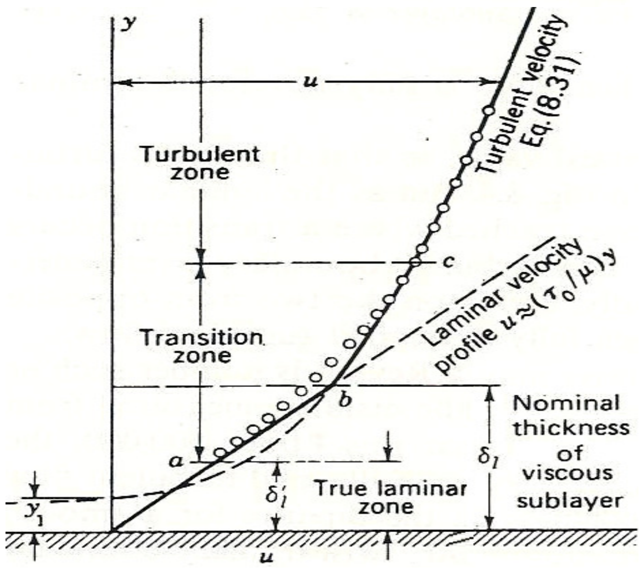


Figure 8.8 Velocity profile near a solid wall (vertical scale greatly exaggerated).

of intersection, but that one curve must merge gradually into the other, as shown by the experimental points.

Any value taken for the thickness of this viscous sublayer must be purely arbitrary. The simultaneous solution of the equations for the two curves, together with some experimental factors, will give the value of y for point b as follows

$$\delta_l = 11.6 \frac{\nu}{\sqrt{\tau_0/\rho}} \tag{8.28}$$

where δ_l is referred to as the *nominal thickness of the viscous sublayer*. The transition curve ac determined by measurements indicates that a is a better limit of the viscous-sublayer thickness. Present information is that the thickness of the viscous sublayer out to point a is approximately

$$\delta_l' = 3.5 \frac{\nu}{\sqrt{\tau_0/\rho}} \tag{8.29}$$

In a circular pipe the laminar velocity profile has been shown to be a parabola, but in this extremely thin region near the wall it can scarcely be distinguished from a straight line.

The transition zone may be said to extend from a to c in Fig. 8.8. For the latter point the value of y has been estimated to be about $70\nu/\sqrt{\tau_0/\rho}$. Beyond this the flow is so turbulent that viscous shear is negligible.

From Eq. (8.15)

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{fV^2}{8}}$$

and making this substitution in Eq. (8.28), we obtain

$$\delta_l = \frac{32.8\nu}{V\sqrt{f}} \tag{8.30}$$

from which it is seen that the higher the velocity or the lower the kinematic viscosity, the thinner the viscous sublayer. Thus, for a given constant pipe diameter, the thickness of the viscous sublayer decreases as the Reynolds number increases.

It is now in order to discuss what is meant by a smooth wall. There is no such thing in reality as a mathematically smooth surface. But if the irregularities on any actual surface are such that the effects of the projections do not pierce through the viscous sublayer (Fig. 8.8), the surface is *hydraulically smooth* from the fluid-mechanics viewpoint. If the effects of the projections extend beyond the sublayer, the laminar layer is broken up and the surface is no longer hydraulically smooth. To be more specific, in Fig. 8.9 if $\delta_l > 6e$, the pipe will behave as though it is hydraulically smooth, while if $\delta_l < 0.3e$, the pipe will behave as *wholly rough*, the significance of which is discussed in Sec. 8.10. In between these values, i.e., with $6e > \delta_l > 0.3e$ the pipe will behave in a transitional mode; that is, neither hydraulically smooth nor wholly rough.

Inasmuch as the thickness of the viscous sublayer in a given pipe will decrease with an increase in Reynolds number, it is seen that the same pipe may be hydraulically smooth at low Reynolds numbers and rough at high Reynolds numbers. Thus, even a relatively smooth pipe may behave as a rough pipe if the Reynolds number is high enough. It is also apparent that, with increasing Reynolds number, there is a gradual transition from smooth to rough pipe flow. These concepts are depicted schematically in Fig. 8.9, where e is the equivalent height of the roughness projection.

8.10 VELOCITY PROFILE IN TURBULENT FLOW

Prandtl reasoned that turbulent flow in a pipe is strongly influenced by the flow phenomena near the wall. In the vicinity of the wall, $\tau \approx \tau_0$. He assumed that the mixing length l near the wall was proportional to the distance from the wall, that

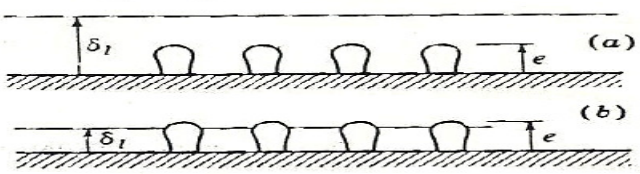


Figure 8.9 Turbulent flow near boundary. (*a*) Relatively low **R**, $\delta_l > e$. If $\delta_l > 6e$ pipe behaves as a smooth pipe. (*b*) Relatively high **R**, $\delta_l < e$. If $\delta_l < 0.3e$ pipe behaves as a wholly rough pipe.

is, $l = Ky$. By experiment it has been determined that K has a value of 0.40.[1] Using these assumptions and applying Eq. (8.27), we get

$$\tau \approx \tau_0 = \rho l^2 \left(\frac{du}{dy}\right)^2 = \rho K^2 y^2 \left(\frac{du}{dy}\right)^2 \quad \text{or} \quad du = \frac{1}{K}\sqrt{\frac{\tau_0}{\rho}}\frac{dy}{y}$$

from which $u = 2.5\sqrt{\dfrac{\tau_0}{\rho}} \ln y + C$

[1] If the fluid is not clear, i.e., if it is carrying particles in suspension, K will have a value less than 0.4

The constant C may be evaluated by noting that $u = u_{max}$ when $y = r_0$. Substituting the expression for C, replacing y by $r_0 - r$, and transforming to log, the equation becomes

$$u = u_{max} - 2.5\sqrt{\frac{\tau_0}{\rho}} \ln \frac{r_0}{r_0 - r} = u_{max} - 5.75\sqrt{\frac{\tau_0}{\rho}} \log \frac{r_0}{r_0 - r} \quad (8.31)$$

Although this equation is derived by assuming certain relations very near to the wall, it has been found to hold practically to the axis of the pipe.

Starting with the derivation of Eq. (8.27), this entire development is open to argument at nearly every step. But the fact remains that Eq. (8.31) agrees very closely with actual measurements of velocity profiles for both smooth and rough pipes. However, there are two zones in which the equation is defective. At the axis of the pipe du/dy must be zero. But Eq. (8.31) is logarithmic and does not have a zero slope at $r = 0$, and hence the equation gives a velocity profile with a sharp point (or cusp) at the axis, whereas in reality it is rounded at the axis. This discrepancy affects only a very small area and involves very slight error in computing the rate of discharge when using Eq. (8.31).

Equation (8.31) is also not applicable very close to the wall. In fact it indicates that when $r = r_0$, the value of u is minus infinity. The equation indicates that $u = 0$, not at the wall, but at a small distance from it, shown as y_1 in Fig. 8.8. However, this discrepancy is well within the confines of the viscous sublayer, where the equation is not supposed to apply. Moreover as the viscous sublayer is very thin, the flow within it has very little effect upon the total rate of discharge.

Hence, although Eq. (8.31) is not perfect, it is reliable except for these two small areas, and thus the rate of discharge may be determined with a high degree of accuracy by using the value of u given by it and integrating over the area of the pipe. Thus

$$Q = \int u\,dA = 2\pi \int_0^{r_0} ur\,dr$$

Substituting the first expression of Eq. (8.31) for u, integrating and dividing by the pipe area πr_0^2, the mean velocity is[1]

$$V = u_{max} - 2.5\sqrt{\frac{\tau_0}{\rho}}\left[\ln r_0 - \frac{2}{r_0^2}\int_0^{r_0} r \ln(r_0 - r)\,dr\right]$$

This equation reduces to

$$V = u_{max} - \frac{3}{2} \times 2.5\sqrt{\frac{\tau_0}{\rho}} = u_{max} - 1.33V\sqrt{f} \quad (8.32)$$

From this last equation the *pipe factor*, which is the ratio of the mean to the maximum velocity, may be obtained. It is

$$\frac{V}{u_{max}} = \frac{1}{1 + 1.33\sqrt{f}} \quad (8.33)$$

Using the relation of Eq. (8.33) in Eq. (8.31) and replacing $\sqrt{\tau_0/\rho}$ by $\sqrt{fV^2/8}$, the result is

$$u = (1 + 1.33\sqrt{f})V - 2.04\sqrt{f}\, V \log \frac{r_0}{r_0 - r} \quad (8.34)$$

which enables a velocity profile to be plotted for any mean velocity and any value of f in turbulent flow. In Fig. 8.10 may be seen profiles for both a smooth and a rough pipe plotted from this equation. The only noticeable difference

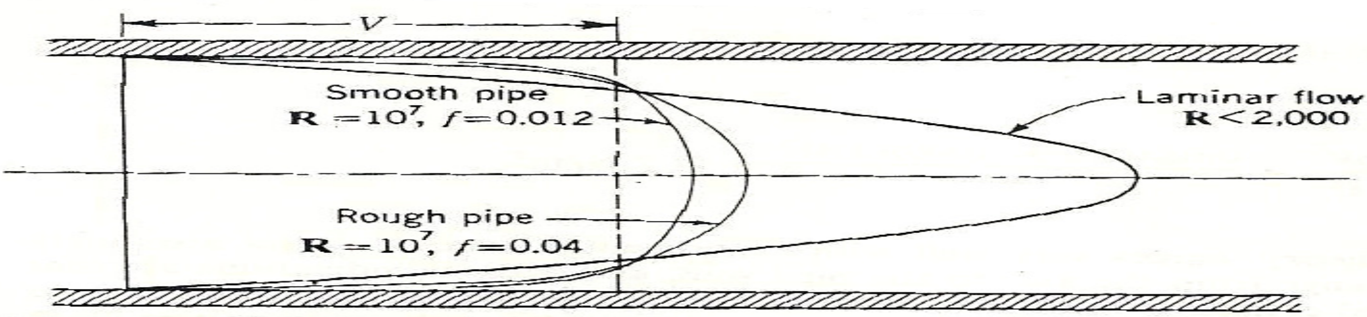


Figure 8.10 Velocity profiles for equal flow rates.

[1] The integral results in indeterminate values at $r = r_0$, as we should expect, inasmuch as the equation for u does not really apply close to the wall. However, these have been shown to reduce to negligible quantities. See B. A. Bakhmeteff, "The Mechanics of Turbulent Flow," p. 70, Princeton University Press, Princeton, N.J., 1941.

between these and measured profiles is that the latter are more rounded at the axis of the pipe.[1]

Comparing the turbulent-flow-velocity profiles with the laminar-flow-velocity profile (Fig. 8.10) shows the turbulent-flow profiles to be much flatter near the central portion of the pipe and steeper near the wall. It is also seen that the turbulent profile for the smooth pipe is flatter near the central section (i.e., blunter) than for the rough pipe. In contrast, the velocity profile in laminar flow is independent of pipe roughness.

As a theoretical equation has now been derived for the velocity profile for turbulent flow in circular pipes, it is also possible to derive equations for the kinetic-energy and momentum-correction factors when mean velocities are used. These equations are[2]

$$\alpha = 1 + 2.7f \qquad (8.35a) \qquad\qquad \beta = 1 + 0.98f \qquad (8.35b)$$

Illustrative Example 8.3 The head loss in 60 m of 15 cm-diameter pipe is known to be 8 m when oil ($s = 0.90$) of viscosity 0.04 N·s/m² flows at 0.06 m³/s. Determine the centerline velocity, the shear stress at the wall of the pipe, and the velocity at 5 cm from the centerline.

The first step is to determine whether the flow is laminar or turbulent.

$$V = \frac{Q}{A} = \frac{0.06}{0.01767} = 3.4 \text{ m/s} \qquad \mathbf{R} = \frac{DV\rho}{\mu} = \frac{0.15(3.4)(0.9 \times 1000)}{0.04} = 11{,}475$$

Since $\mathbf{R} > 2{,}000$, the flow is turbulent. Using Eq. (8.12), the friction factor can be found:

$$f = \frac{h_L D(2g)}{LV^2} = \frac{8(0.15)(19.62)}{60(3.4^2)} = 0.034$$

From Eq. (8.33), $\quad u_{max} = V(1 + 1.33\sqrt{f}) = 3.4(1 + 1.33\sqrt{0.034}) = 4.2$ m/s

Equation (8.15) yields $\quad \tau_0 = \dfrac{f\rho V^2}{8} = \dfrac{0.034(0.9 \times 1000)(3.4)^2}{8} = 44.2$ N/m²

[1] Although the preceding theory agrees very well with experimental data, it is not absolutely correct throughout the entire range from the axis to the pipe wall, and present indications are that some slight shift in the numerical constants will agree somewhat more closely with test data. Thus, in Eqs. (8.33) and (8.34) the 1.33 may be replaced by 1.44, and in Eq. (8.34), although many writers use 2 instead of 2.04, a better practical value seems to be 2.15.

[2] L. F. Moody, Some Pipe Characteristics of Engineering Interest, *Houille Blanche*, May–June, 1950.

Finally, from Eq. (8.31),

$$u_{5\,cm} = 4.2 - 5.75\sqrt{\frac{44.2}{0.9 \times 1000}}\log\left(\frac{7.5}{7.5 - 5}\right) \qquad u_{5\,cm} = 4.2 - 1.4 = 2.8 \text{ m/s}$$

Note that if the flow had been laminar, the velocity profile would have been parabolic and th centerline velocity would have been twice the average velocity.

8.11 PIPE ROUGHNESS

Unfortunately, there is as yet no scientific way of measuring or specifying the roughness of commercial pipes. Several experimenters have worked with pipes with artificial roughness produced by various means so that the roughness could be measured and described by geometrical factors, and it has been proved that the friction is dependent not only upon the size and shape of the projections, but also upon their distribution or spacing. Much remains to be done before this problem is completely solved.

The most noteworthy efforts in this direction were made by a German engineer Nikuradse, a student of Prandtl's. He coated several different sizes of pipe with sand grains which had been segregated by sieving so as to obtain different sizes of grain of reasonably uniform diameters. The diameters of the sand grains may be represented by e, which is known as the *absolute roughness*. In Sec. 8.4 dimensional analysis of pipe flow showed that for a smooth-walled pipe the friction factor f is a function of Reynolds number. A general approach, including e as a parameter, reveals that $f = \phi(\mathbf{R}, e/D)$. The term e/D is known as the *relative roughness*. In his experimental work Nikuradse had values of e/D ranging from 0.000985 to 0.0333.

In the case of artificial roughness such as this, the roughness is uniform whereas in commercial pipes it is irregular both in size and in distribution However, the roughness of commercial pipe may be described by e, which means that the pipe has the same value of f at a high Reynolds number that would be obtained if a smooth pipe were coated with sand grains of uniform size e.

For pipes it has been found that if $\delta_l > 6e$, the viscous sublayer completely submerges the effect of e. Von Kármán, using information from Eq. (8.31) and data from Nikuradse's experiments, developed an equation for friction factor for such a case: "Smooth-pipe" flow $\delta_l > 6e$:

$$\frac{1}{\sqrt{f}} = 2\log \mathbf{R}\sqrt{f} - 0.8 \qquad (8.36)$$

This equation applies to any pipe as long as $\delta_l > 6e$; when this condition prevails, the flow is known as *smooth flow*. The equation has been found to be reliable for smooth pipes for all values of $\mathbf{R}$ over 4,000. For such pipes, i.e., drawn tubing, brass, glass, etc., it can be extrapolated with confidence for values of $\mathbf{R}$ fa beyond any present experimental values because it is functionally correct, assuming wall surface so smooth that the effects of the projections do not pierce the viscous sublayer, which becomes increasingly thinner with increasing **R**. That this is so is evident from the fact that the formula yields a value of $f = 0$ for $\mathbf{R} = \infty$. This is in accord with the facts because **R** is infinite for a fluid of zero viscosity, and for such a case f must be zero.

Blasius[1] has shown that for Reynolds numbers between 3,000 and 100,000 the friction factor for a *very smooth pipe* may be expressed approximately as

$$f = \frac{0.316}{\mathbf{R}^{0.25}} \tag{8.37}$$

He also found that over this range of Reynolds numbers the velocity profile in a smooth pipe is closely approximated by the expression

$$\frac{u}{u_{\max}} = \left(\frac{y}{r_0}\right)^{1/7} \tag{8.38}$$

where $y = r_0 - r$, the distance from the pipe wall. This equation is commonly referred to as the *seventh-root law* for turbulent-velocity distribution. Though it is not absolutely accurate, it is useful because it is easy to work with mathematically. At Reynolds numbers above 100,000 a somewhat smaller exponent must be used to give good results.

At high Reynolds numbers δ_l becomes smaller. If $\delta_l < 0.3e$, it has been found that the pipe behaves as a *wholly rough* pipe; i.e., its friction factor is independent of the Reynolds number. For such a case von Kármán found that the friction factor could be expressed as

"Rough-pipe" flow $\delta_l < 0.3e$:

$$\frac{1}{\sqrt{f}} = 2 \log \frac{D}{e} + 1.14 \tag{8.39}$$

The values of f from this equation correspond to the values from the chart (Fig. 8.11), where the lines become horizontal.

In the gap where $6e > \delta_l > 0.3e$ neither smooth flow [Eq. (8.36)] nor wholly rough flow [Eq. (8.39)] applies. Colebrook[2] found that in this intermediate range an approximate relationship was

Transitional flow $6e > \delta_l > 0.3e$:

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathbf{R}\sqrt{f}}\right) \tag{8.40}$$

[1] H. Blasius, Das Ähnlichkeitsgesetz bei Reibungsvorgängen in Flüssigkeiten, *Forsch. Gebiete Ingenieurw.*, vol. 131, 1913.

[2] C. F. Colebrook, Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws, *J. Inst. Civil Engrs.* (*London*), February, 1939.

8.12 CHART FOR FRICTION FACTOR

As the preceding equations for f are very inconvenient to use, it is preferable to obtain numerical values from a chart,[1] such as Fig. 8.11, prepared by Moody. This chart is based on the best information available and has been plotted with the aid of the equations of the preceding section. As a matter of convenience, values for air and water at 15°C have been placed at the top of the chart to save the necessity of computing Reynolds number for those two typical cases.

The chart shows that there are four zones: laminar flow; a critical range where values are uncertain because the flow might be either laminar or turbulent; a transition zone, where f is a function of both Reynolds number and relative pipe roughness; and a zone of complete turbulence (*rough pipe flow*) where the value of f is independent of Reynolds number and depends solely upon the relative roughness.

There is no sharp line of demarcation between the transition zone and the zone of complete turbulence. The dashed line of Fig. 8.11 that separates the two zones was suggested by R. J. S. Pigott; the equation of this line is $\mathbf{R} = 3500/(e/D)$.

For use with this chart, values of e may be obtained from Table 8.1. As the ratio e/D is dimensionless, any units may be used provided they are the same for

Table 8.1 Values of absolute roughness e for new pipes

	Millimeters		Millimeters
Drawn tubing, brass, lead, glass, centrifugally spun cement, bituminous lining, transite	0.0015	Wood stave	0.18 to 0.9
Commercial steel or wrought iron	0.046	Concrete	0.3 to 3
Welded-steel pipe	0.046	Riveted steel	0.9 to 9
Asphalt-dipped cast iron	0.12	Cast iron, average	0.25
		Galvanized iron	0.15

Note: $\frac{e}{D} = \frac{e \text{ in mm}}{D \text{ in mm}} = 10^{-1} \times \frac{e \text{ in mm}}{D \text{ in cm}}$

[1] Fig. 8.11 is often referred to as a *Stanton diagram* as Stanton was the first person to propose such a plot.

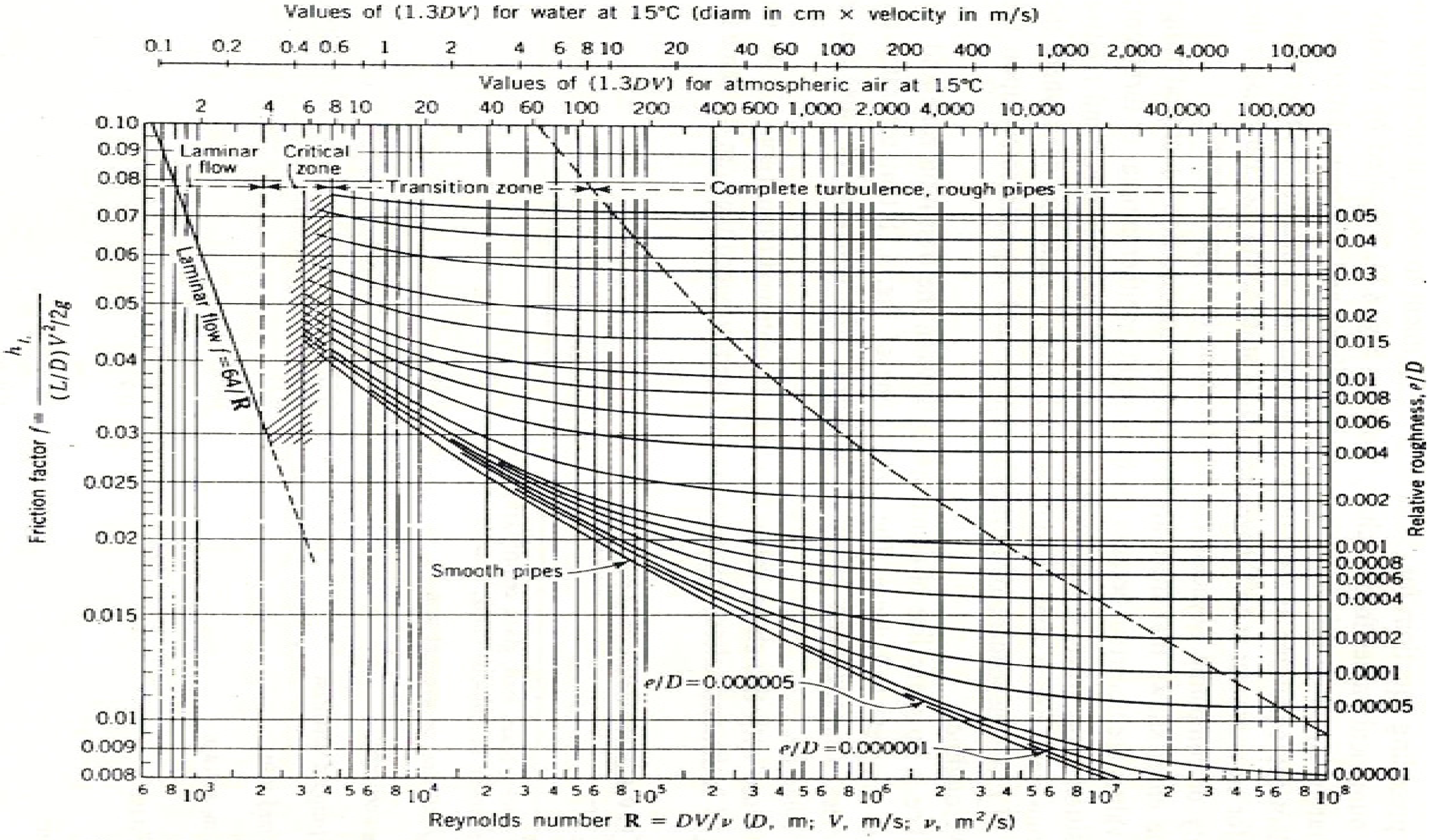


Figure 8.11 Friction factor for pipes (Moody diagram).

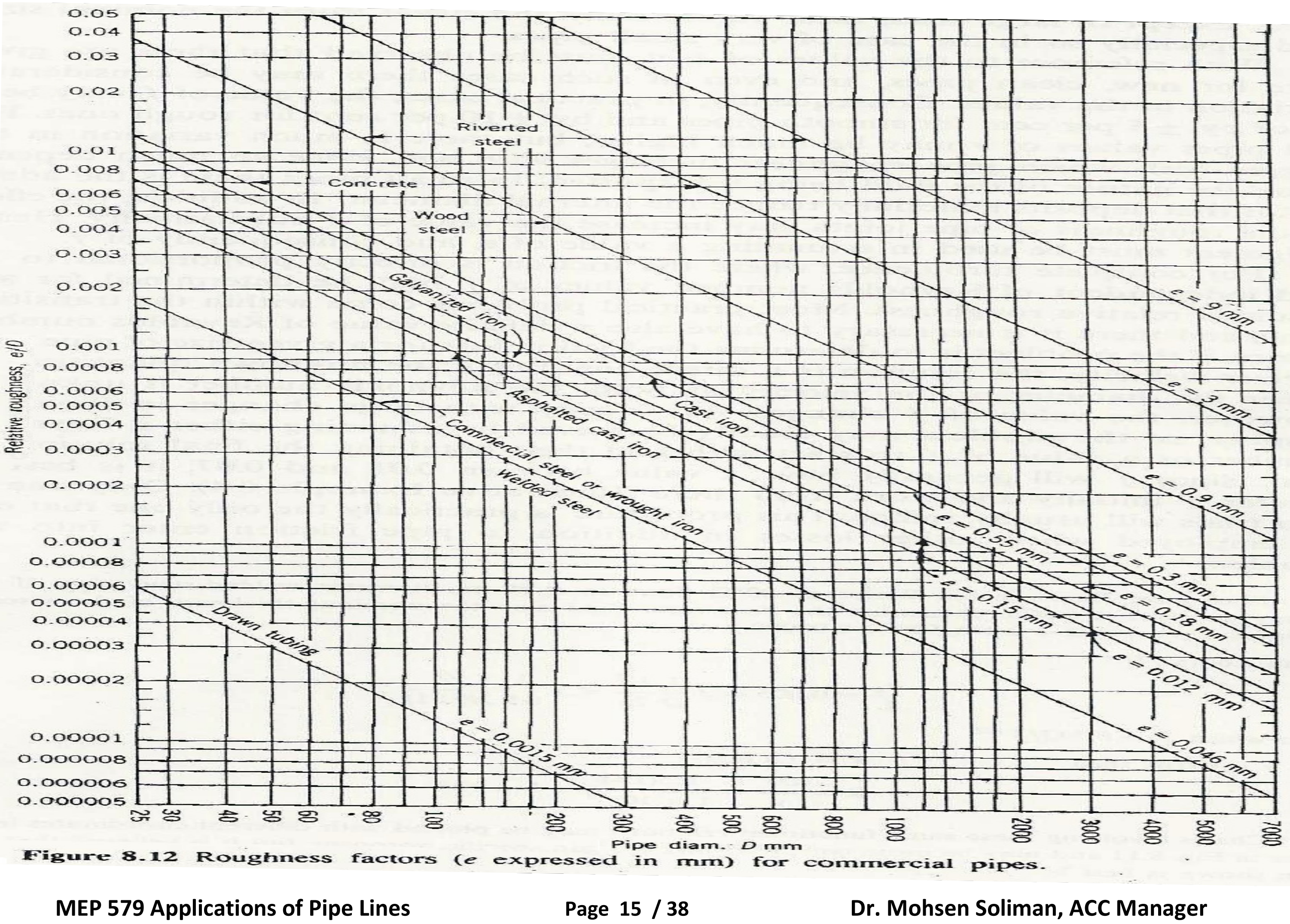


Figure 8.12 Roughness factors (*e* expressed in mm) for commercial pipes.

both. Values of e/D for commercial pipe may conveniently be found from Fig. 8.12, which has also been prepared by Moody. In the use of these charts, as well as in Eq. (8.13), the exact value of the internal diameter of the pipe should be used. Except in large sizes, these values differ somewhat from the nominal sizes, and especially so in the case of very small pipes.

With reference to the values of e, it must be observed that these are given here for new, clean pipes, and even in such cases there may be considerable variation in the values. Consequently, in practical cases, the value of f may be in error by ± 5 per cent for smooth pipes and by ± 10 per cent for rough ones. For old pipes values of e may be much higher, but there is much variation in the degree with which pipe roughness increases with age, since so much depends upon the nature of the fluid being transported. In small pipes there is the added factor that deposits materially reduce the internal diameter. In addition, the effect of the roughness of pipe joints may increase the value of f substantially. Hence judgment must be used in estimating a value of e, and consequently of f.

For complete turbulence, where the friction is directly proportional to V^2 and independent of Reynolds number, values of f may be determined for any assumed relative roughness. Most practical problems come within the transition zone, and there it is necessary to have also a definite value of Reynolds number. Hence, if the problem is to determine the friction loss for a given size of pipe with a given velocity, the solution is a direct one. But if the unknown quantities are either the diameter or the velocity or both, the Reynolds number is unknown. However, the value of f changes very slowly with large changes in Reynolds number; so the problem may readily be solved by assuming either a Reynolds number or a value of f to start with and then obtaining the final solution by trial. Since f will generally have a value between 0.01 and 0.07, it is best to assume f initially and work from there (Illustrative Example 8.4). Only one or two trials will usually suffice. This procedure is practically the only one that can be employed where other losses in addition to pipe friction enter into the problem.[1]

[1] Charts involving these same functional relations may be plotted with different coordinates from those in Fig. 8.11 and may be more convenient for certain specific purposes, but it is believed that the form shown is best both for instruction purposes and for general use.

Illustrative Example 8.4 Water at 20°C flows in a 50-cm-diameter welded-steel pipe. If the energy gradient is 0.006, determine the flow rate. Find also the nominal thickness of the viscous sublayer. (*Note*: $e/D = 0.046/500 = 0.00009$.)

From Eq. (8.12) $\quad \dfrac{h_L}{L} = 0.006 = f\dfrac{1}{D}\dfrac{V^2}{2g} = f\dfrac{1}{0.5}\dfrac{V^2}{2(9.81)}$ from which $V = 0.243/f^{1/2}$.

Try $f = 0.030$, then $\quad V = 1.4$ m/s and $\quad \mathbf{R} = \dfrac{DV}{\nu} = \dfrac{0.5(1.4)}{1 \times 10^{-6}} = 7 \times 10^5$

For $\mathbf{R} = 7 \times 10^5$ and $e/D = 0.00009$ the pipe friction chart (Fig. 8.11) indicates $f = 0.0136$. Since the f versus $\mathbf{R}$ curve is relatively flat, we will assume $f = 0.0136$ for the next trial. For this case, $V = 0.243/f^{1/2} = 2.08$ m/s and $\mathbf{R} \approx 10^6$ For $R = 10^6$ the chart indicates $f = 0.0131$.

For the next trial, let $f = 0.0131$. This gives $V = 2.12$ m/s and $\mathbf{R}$ is still $\approx 10^6$, hence $V = 2.12$ m/s is the answer.

$$Q = AV = \frac{\pi(0.5)^2}{4}(2.12) = 0.416 \text{ m}^3/\text{s}$$

Eq. (8.30) $\quad \delta_l = \dfrac{32.8\nu}{V\sqrt{f}} = \dfrac{32.8(10^{-6}\text{ m}^2/\text{s})}{2.12\text{ m/s}\sqrt{0.0131}} \qquad \delta_l = 135 \times 10^{-6}\text{ m} = 0.135\text{ mm}$

Note $\delta_l = 2.9e$, therefore the flow is in the transition zone which is typical.

8.13 FLUID FRICTION IN NONCIRCULAR CONDUITS

Most closed conduits used in engineering practice are of circular cross section; however, rectangular ducts and cross sections of other geometry are occasionally used. Some of the foregoing equations may be modified for application to noncircular sections by use of the hydraulic-radius concept.

The hydraulic radius was defined (Sec. 8.3) as $R_h = A/P$, where A is the cross-sectional area and P is the wetted perimeter. For a circular pipe flowing full,

$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} \qquad (8.41) \qquad \text{or} \qquad D = 4R_h \qquad (8.42)$$

These values may be substituted into Eq. (8.13) and into the expression for Reynolds number. Thus

$$h_L = \frac{f}{4}\frac{L}{R_h}\frac{V^2}{2g} \qquad (8.43) \qquad \mathbf{R} = \frac{(4R_h)V\rho}{\mu} \qquad (8.44)$$

From these two expressions the head loss in noncircular conduits can be estimated by use of Fig. 8.11, where e/D is replaced by $e/4R_h$. This approach gives good results for turbulent flow, but for laminar flow the results are poor, because in such flow frictional phenomena are caused by viscous action throughout the body of the fluid, while in turbulent flow the frictional effect is accounted for largely by the region close to the wall; i.e., it depends on the wetted perimeter.

8.14 EMPIRICAL EQUATIONS FOR PIPE FLOW

The presentation of friction loss in pipes given in Secs. 8.1 to 8.12 incorporates the best knowledge available on this subject, as far as application to Newtonian fluids (Sec. 1.11) is concerned. Admittedly, however, the trial-and-error type of solution, especially when encumbered with computations for relative roughness and Reynolds number, becomes tedious when repeated often for similar conditions, as with a single fluid such as water. It is natural, therefore, that empirical design formulas have been developed, applicable only to specific fluids and conditions but very convenient in a certain range. Perhaps the best example of such a formula is that of Hazen and Williams, applicable only to the flow of water in pipes larger than 5 cm and at velocities less than 3 m/s, but widely used in the waterworks industry. This formula is given in the form

English units:
$$V = 0.85 C_{HW} R_h^{0.63} S^{0.54} \tag{8.45}$$

where R_h (m) is the hydraulic radius (Sec. 8.3), and $S = h_L/L$, the energy gradient. The advantage of this formula over the standard pipe-friction formula is that the roughness coefficient C_{HW} is not a function of the Reynolds number and trial solutions are therefore eliminated. Values of C_{HW} range from 140 for very smooth, straight pipe down to 110 for new riveted-steel and vitrified pipe and to 90 or 80 for old and tuberculated pipe.

Another empirical formula, which is discussed in detail in Sec. 11.3, is the Manning formula, which is
$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{8.46}$$

where n is a roughness coefficient, varying from 0.009 for the smoothest brass or glass pipe, to 0.014 for average drainage tile or vitrified sewer pipe, to 0.021 for corrugated metal, and up to 0.035 for tuberculated cast-iron pipe (Table 11.1). The Manning formula applies to about the same flow range as does the Hazen-Williams formula.

Nomographic charts and diagrams have been developed for the application of Eqs. (8.45) and (8.46). The attendant lack of accuracy in using these formulas is not important in the design of water-distribution systems, since it is seldom possible to predict the capacity requirements with high precision.

8.15 MINOR LOSSES IN TURBULENT FLOW

Losses due to the *local* disturbances of the flow in conduits such as changes in cross section, projecting gaskets, elbows, valves, and similar items are called *minor losses*. In the case of a very long pipe or channel, these losses are usually insignificant in comparison with the fluid friction in the length considered. But if the length of pipe or channel is very short, these so-called minor losses may actually be major losses. Thus, in the case of the suction pipe of a pump, the loss of head at entrance, especially if a strainer and a foot valve are installed, may be very much greater than the friction loss in the short inlet pipe.

Whenever the velocity of a flowing stream is altered either in direction or in magnitude in turbulent flow, eddy currents are set up and a loss of energy in excess of the pipe friction in that same length is created.[1] Head loss in decelerating (i.e., diverging) flow is much larger than that in accelerating (i.e., converging) flow (Sec. 8.19). In addition, head loss generally increases with an increase in the geometric distortion of the flow. Though minor losses are usually confined to a very short length of path, the effects may not disappear for a considerable distance downstream. Thus an elbow in a pipe may occupy only a small length but the disturbance in the flow will extend for a long distance downstream.

The most common sources of minor loss are described in the remainder of this chapter. Such losses may be represented in one of two ways. They may be expressed as $kV^2/2g$, where k must be determined for each case, or they may be represented as being equivalent to a certain length of straight pipe, usually expressed in terms of the number of pipe diameters.

8.16 LOSS OF HEAD AT ENTRANCE

Referring to Fig. 8.13, it may be seen that, as fluid from the reservoir enters the pipe, the streamlines tend to converge, much as though this were a jet issuing from a sharp-edged orifice, so that at B a maximum velocity and a minimum pressure are found.[2] At B the central stream is surrounded by fluid which is in a state of turbulence but has very little forward motion. Between B and C the fluid is in a very disturbed condition because the stream expands and the velocity decreases while the pressure rises. From C to D the flow is normal.

It is seen that the loss of energy at entrance is distributed along the length AC, a distance of several diameters. The increased turbulence and vortex motion in this portion of the pipe cause the friction loss to be much greater than in a corresponding length where the flow is normal, as is shown by the drop of the total-energy line. Of this total loss, a portion h' would be due to the normal pipe friction. Hence the difference between this and the total, or h'_e, is the true value of the extra loss caused at entrance. The loss of head at entrance may be expressed as
$$h'_e = k_e \frac{V^2}{2g} \tag{8.47}$$

[1] In laminar flow these losses are insignificant because irregularities in the flow boundary create a minimal disturbance to the flow and separation is essentially nonexistent.

[2] Section B, the point of maximum contraction of the flow, is referred to as the *vena contracta*.

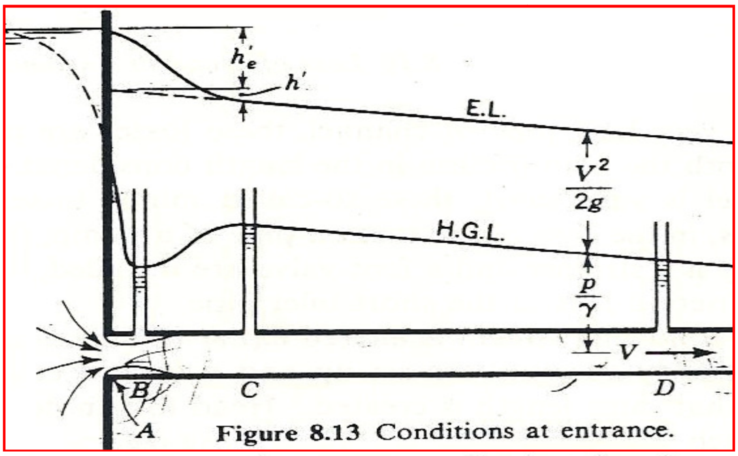


Figure 8.13 Conditions at entrance.

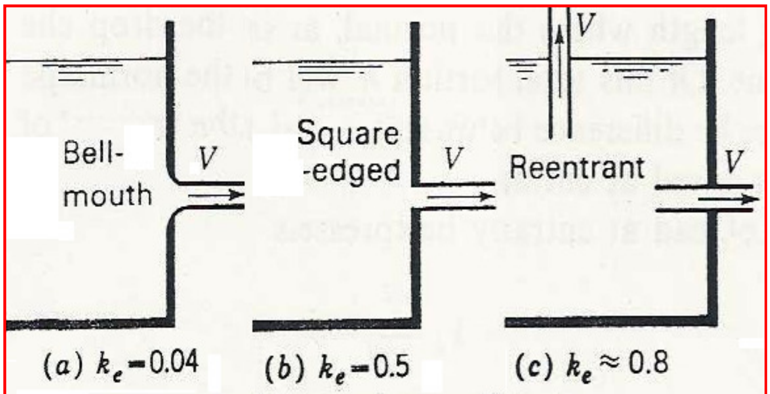


Figure 8.14 Entrance loss coefficients.

where V is the mean velocity in the pipe, and k_e is the loss coefficient whose general values are shown in Fig. 8.14.

The entrance loss is caused primarily by the turbulence created by the enlargement of the stream after it passes section B, and this enlargement in turn depends upon how much the stream contracts as it enters the pipe. Thus it is very much affected by the conditions at the entrance to the pipe. Values of the entrance-loss coefficients have been determined experimentally. If the entrance to the pipe is well rounded or *bell-mouthed* (Fig. 8.14*a*), there is no contraction of the stream entering and the coefficient of loss is correspondingly small. For a *flush* or *square-edged entrance*, such as shown in Fig. 8.14*b*, k_e has a value of about 0.5. A *reentrant tube*, such as shown in Fig. 8.14*c*, produces a maximum contraction of the entering stream because the streamlines come from around the outside wall of the pipe, as well as more directly from the fluid in front of the entrance. The degree of the contraction depends upon how far the pipe may project within the reservoir and also upon how thick the pipe walls are, compared with its diameter. With very thick walls, the conditions approach that of a square-edged entrance. For these reasons the loss coefficients for reentrant tubes vary; for very thin tubes $k_e \approx 0.8$.

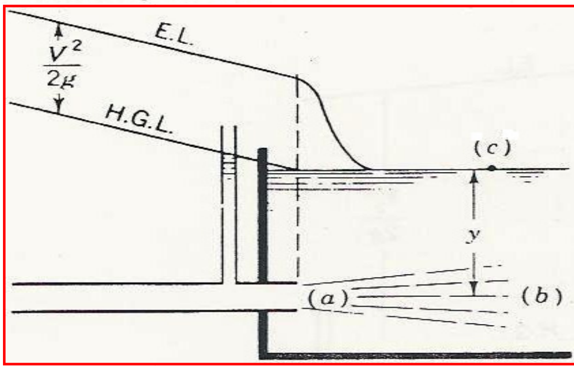


Figure 8.15 Submerged discharge loss.

8.17 LOSS OF HEAD AT SUBMERGED DISCHARGE[1]

When a fluid with a velocity V is discharged from the end of a pipe into a closed tank or reservoir which is so large that the velocity within it is negligible, the entire kinetic energy of the flow is dissipated. Hence the discharge loss is

$$h'_d = \frac{V^2}{2g} \tag{8.48}$$

That this is true may be shown by writing an energy equation between (a) and (c) in Fig. 8.15. Taking the datum plane through (a) and recognizing that the pressure head of the fluid at (a) is y, its depth below the surface, $H_a = y + 0 + V^2/2g$ and $H_c = 0 + y + 0$. Therefore

$$h'_d = H_a - H_c = \frac{V^2}{2g}$$

The discharge loss coefficient is 1.0 under all conditions; hence the only way to reduce the discharge loss is to reduce the value of V by means of a diverging tube. This is the reason for diverging draft tube from reaction turbine (Sec.16.6).

As contrasted with entrance loss, it must here be emphasized that discharge loss occurs *after* the fluid *leaves* the pipe,[2] while entrance loss occurs *after* the fluid *enters* the pipe.

[1] This topic was first discussed in Sec. 4.13.

[2] In a short pipe where the discharge loss may be a major factor, greater accuracy is obtained by using the correction factor α, as explained in Sec. 4.1 [see also Eq. (8.35*a*)].

8.18 LOSS DUE TO CONTRACTION

Sudden Contraction

The phenomena attending the sudden contraction of a flow are shown in Fig 8.16. There is a marked drop in pressure due to the increase in velocity and to the loss of energy in turbulence. It is noted that in the corner upstream at section C

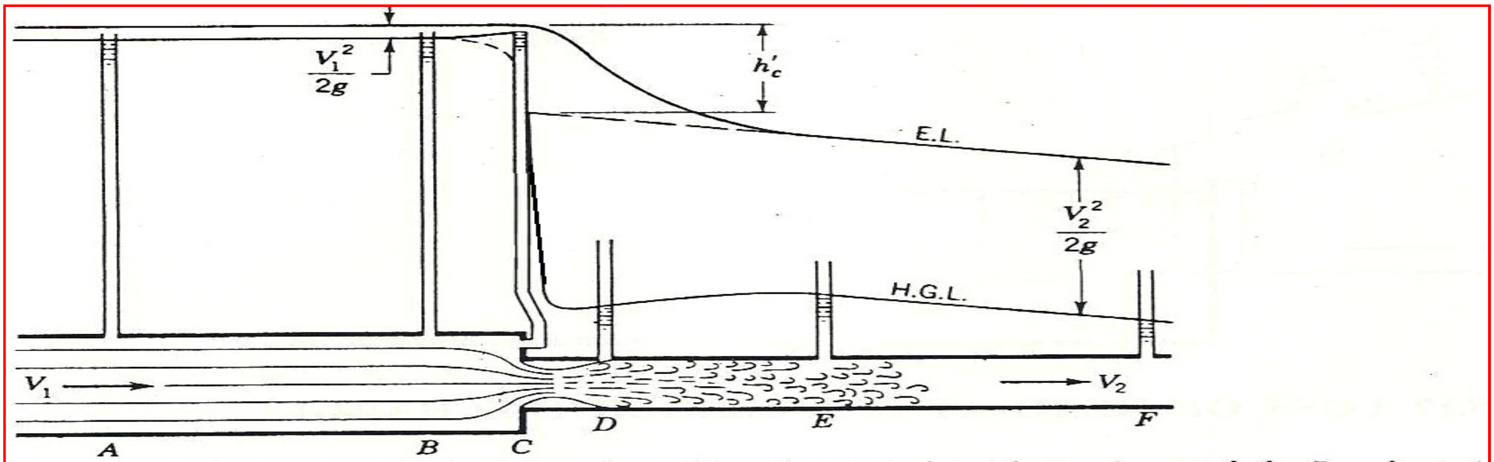


Figure 8.16 Loss due to sudden contraction. *(Plotted to scale from observations made by Daugherty.)*

there is a rise in pressure because the streamlines here are curving, so that the centrifugal action causes the pressure at the pipe wall to be greater than in the center of the stream. The dashed line indicates the pressure variation along the centerline streamline from sections *B* to *C*.

From *C* to *E* the conditions are similar to those described for entrance. The loss of head for a sudden contraction may be represented by

$$h_c' = k_c \frac{V_2^2}{2g} \tag{8.49}$$

where k_c has the values given in Table 8.2.

The entrance loss of Sec. 8.16 is a special case where $D_2/D_1 = 0$.

Gradual Contraction

In order to reduce the foregoing losses, abrupt changes of cross section should be avoided. This may be accomplished by changing from one diameter to the other by means of a smoothly curved transition or by employing the frustum of a cone. With a smoothly curved transition a loss coefficient k_c as small as 0.05 is possible. For conical reducers a minimum k_c of about 0.10 is obtained with a total cone angle of 20 to 40°. Smaller or larger total cone angles result in higher values of k_c.

Table 8.2 Loss coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

The nozzle at the end of a pipeline (Fig. 8.23*b*) is a special case of gradual contraction. The head loss through a nozzle at the end of a pipeline is given by Eq. (8.49) where k_c is the nozzle loss coefficient whose value commonly ranges from 0.04 to 0.20 and V_j is the jet velocity.[1] The head loss through a nozzle cannot be regarded as a minor loss because the jet velocity head is usually quite large. More details on the flow through nozzles is presented in Sec. 12.6.

8.19 LOSS DUE TO EXPANSION

Sudden Expansion

The conditions at a sudden expansion are shown in Fig. 8.17. There is a rise in pressure because of the decrease in velocity, but this rise is not so great as it would be if it were not for the loss in energy. There is a state of excessive turbulence from *C* to *F* beyond which the flow is normal. The drop in pressure just beyond section *C*, which was measured by a piezometer not shown in the illustration, is due to the fact that the pressures at the wall of the pipe are in this case less than those in the center of the pipe because of centrifugal effects.

Figures 8.16 and 8.17 are both drawn to scale from test measurements for the same diameter ratios and the same velocities and show that the loss due to sudden expansion is greater than the loss due to a corresponding contraction. This is so because of the inherent instability of flow in an expansion where the diverging paths of the flow tend to encourage the formation of eddies within the flow. Moreover, separation of the flow from the wall of the conduit induces

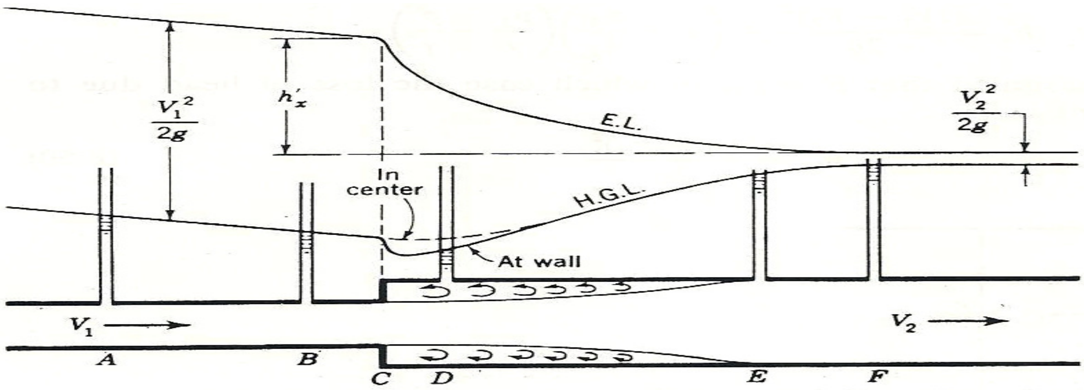


Figure 8.17 Loss due to sudden enlargement. *(Plotted to scale from observations by Daugherty. Velocity the same as in Fig. 8.16.)* [1] See also Eq. (12.13).

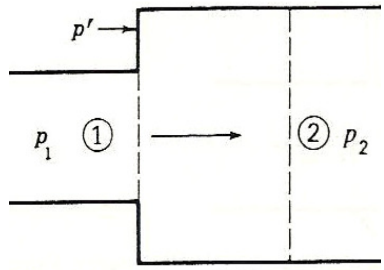


Figure 8.18

pockets of eddying turbulence outside the flow region. In converging flow there is a dampening effect on eddy formation and the conversion from pressure energy to kinetic energy is quite efficient.

An expression for the loss of head in a sudden enlargement can be derived as follows. In Fig. 8.18, section 2 corresponds to section F in Fig. 8.17, which is a section where the velocity profile has become normal again and marks the end of the region of excess energy loss due to the turbulence created by the sudden enlargement. In Fig. 8.18 assume that the pressure at section 2 in the ideal case without friction is p_o. Then in this ideal case $\frac{p_o}{\gamma} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$

If in the actual case the pressure at section 2 is p_2 while the average pressure on the annular ring is p', then, equating the resultant force on the body of fluid between sections 1 and 2 to the time rate of change of momentum between sections 1 and 2, we obtain

$$p_1A_1 + p'(A_2 - A_1) - p_2A_2 = \frac{\gamma}{g}(A_2V_2^2 - A_1V_1^2)$$

From this

$$\frac{p_2}{\gamma} = \frac{A_1}{A_2}\frac{p_1}{\gamma} + \frac{A_2 - A_1}{A_2}\frac{p'}{\gamma} + \frac{A_1}{A_2}\frac{V_1^2}{g} - \frac{V_2^2}{g}$$

The loss of head is given by the difference between the ideal and actual pressure heads at section 2. Thus $h'_x = (p_o - p_2)/\gamma$, and noting that $A_1V_1 = A_2V_2$ and that $A_1V_1^2 = A_1V_1V_1 = A_2V_2V_1$, we obtain, from substituting the above expressions for p_o/γ and p_2/γ into $(p_o - p_2)/\gamma$,

$$h'_x = \frac{(V_1 - V_2)^2}{2g} + \left(1 - \frac{A_1}{A_2}\right)\left(\frac{p_1}{\gamma} - \frac{p'}{\gamma}\right)$$

It is usually assumed that $p' = p_1$, in which case the loss of head due to sudden enlargement is

$$h'_x = \frac{(V_1 - V_2)^2}{2g} \tag{8.50}$$

Although it is possible that under some conditions p' will equal p_1, it is also possible for it to be either more or less than that value, in which case the loss of head will be either less or more than that given by Eq. (8.50). The exact value of p' will depend upon the manner in which the fluid eddies around in the corner adjacent to this annular ring. However, the deviation from Eq. (8.50) is quite small and of negligible importance.

The discharge loss of Sec. 8.17 is seen to be a special case where A_2 is infinite compared with A_1, or $V_2 = 0$, so that Eq. (8.50) will reduce to Eq. (8.48).

Gradual Expansion

To minimize the loss accompanying a reduction in velocity, a diffuser such as shown in Fig. 8.19 may be used. The diffuser may be given a curved outline, or it may be a frustum of a cone. In Fig. 8.19 the loss of head will be some function of the angle of divergence and also of the ratio of the two areas, the length of the diffuser being determined by these two variables.

In flow through a diffuser the total loss may be considered as made up of two factors. One is the ordinary pipe friction loss, which may be represented by

$$h_L = \int \frac{f}{D}\frac{V^2}{2g}\,dL$$

In order to integrate the foregoing, it is necessary to express the variables f, D, and V as functions of L. For our present purpose it is sufficient, however, merely to note that the friction loss increases with the length of the cone. Hence, for given values of D_1 and D_2, the larger the angle of the cone, the less its length and the less the pipe friction, which is indicated by the curve marked F in Fig. 8.20*a*. However, in flow through a diffuser, there is an additional turbulence loss set up by induced currents which produce a vortex motion over and above that which normally exists. This additional turbulence loss will naturally increase with the degree of divergence, as is indicated by the curve marked T in Fig. 8.20*a*, and if the rate of divergence is great enough, there may be a separation at the walls and eddies flowing backward along the walls. The total loss in the diverging cone is then represented by the sum of these two losses, marked k'. This is seen to have a minimum value at 6° for the particular case chosen, which is for a very smooth

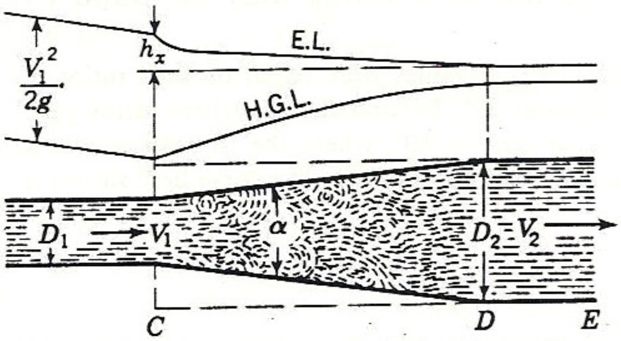


Figure 8.19 Loss due to gradual enlargement.

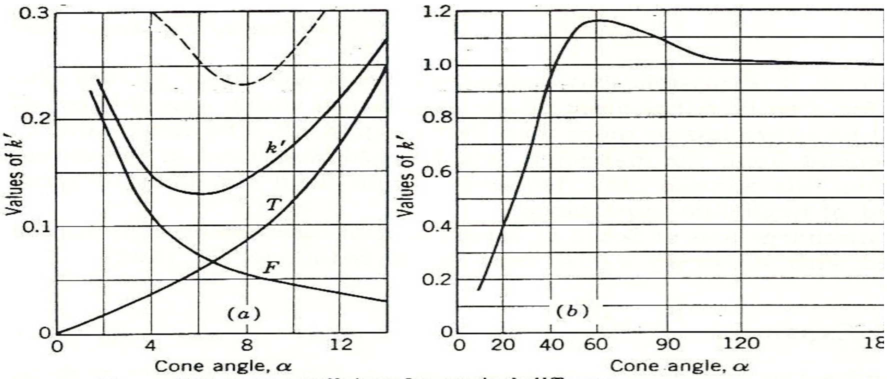


Figure 8.20 Loss coefficient for conical diffusers.

surface. If the surface were rougher, the value of the friction F would be increased. This increases the value of k', which is indicated by the dotted curve, and also shifts the angle for minimum loss to 8°. Thus the best angle of divergence increases with the roughness of the surface.

It has been seen that the loss due to a sudden enlargement is very nearly represented by $(V_1 - V_2)^2/2g$. The loss due to a gradual enlargement is expressed as

$$h' = k' \frac{(V_1 - V_2)^2}{2g} \tag{8.51}$$

Values of k' as a function of the cone angle α are shown in Fig. 8.20*b*,[1] for a wider range than appears in Fig. 8.20*a*. It is of interest to note that at an angle slightly above 40° the loss is the same as that for a sudden enlargement, which is 180° and that between these two the loss is greater than for a sudden enlargement being a maximum at about 60°. This is because the induced currents set up are greater within this range.

8.20 LOSS IN PIPE FITTINGS

The loss of head in pipe fittings may be expressed as $kV^2/2g$, where V is the velocity in a pipe of the nominal size of the fitting. Typical values of k are given in Table 8.3. As an alternative, the head loss due to a fitting may be found by

[1] A. H. Gibson, *Engineering (London)*, Feb. 16, 1912. These values were based on area ratios of 1:9, 1:4, 1:2.25 and gave one curve up to an angle of about 30°. Beyond that the three ratios gave three curves which differed by as much as about 18 per cent at 60°, where the turbulence was a predominating factor, and then drew together again as 180° was approached. The curve here shown is a composite of these three.

Table 8.3 Values of loss factors for pipe fittings*

Fitting	k	L/D	Fitting	k	L/D
Globe valve, wide open	10	350	Medium-radius elbow	0.75	27
Angle valve, wide open	5	175	Long-radius elbow	0.60	20
Close-return bend	2.2	75	45° elbow	0.42	15
T, through side outlet	1.8	67	Gate valve, wide open	0.19	7
Short-radius elbow	0.9	32	half open	2.06	72

* Flow of Fluids through Valves, Fittings, and Pipe, *Crane Co., Tech. Paper* 409, May, 1942. Values based on tests by Crane Co. and at the University of Wisconsin, the University of Texas, and Texas College.

increasing the pipe length by using values of L/D given in the table. However, it must be recognized that these fittings create so much turbulence that the loss caused by them is proportional to V^2, and hence this latter method should be restricted to the case where the pipe friction itself is in the zone of complete turbulence. For very smooth pipes, it is better to use the k values when determining the loss through fittings.

8.21 LOSS IN BENDS AND ELBOWS

In flow around a bend or elbow, because of centrifugal effects [Eq. (4.35)], there is an increase in pressure along the outer wall and a decrease in pressure along the inner wall. The centrifugal force on a number of fluid particles, each of mass m, along the diameter CD of the pipe that is normal to the plane of curvature of the pipe is shown in Fig. 8.21. The centrifugal force on the particles near the center of the pipe, where the velocities are high, is larger than the centrifugal force on the particles near the walls of the pipe, where the velocities are low. Because of this unbalanced condition a secondary flow[1] develops as shown in Fig. 8.21. This combines with the axial velocity to form a double spiral flow which persists for some distance. Thus not only is there some loss of energy within the bend itself, but this distorted flow condition persists for some distance downstream until dissipated by viscous friction. The velocity in the pipe may not become normal again within as much as 100 pipe diameters downstream from the bend. In fact, more than half the friction loss produced by a bend or elbow takes place in the straight pipe following it.

[1] Secondary flow in the bends of open channels is discussed in Sec. 11.21.

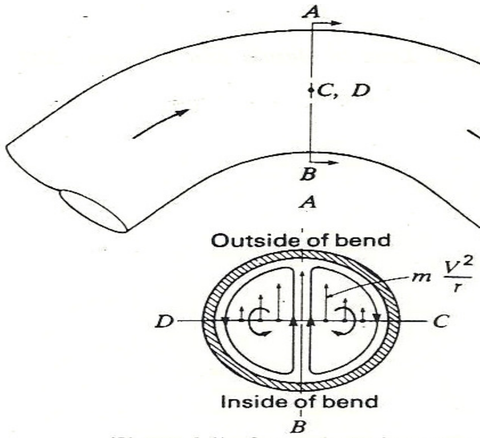


Figure 8.21 Secondary flow in bend.

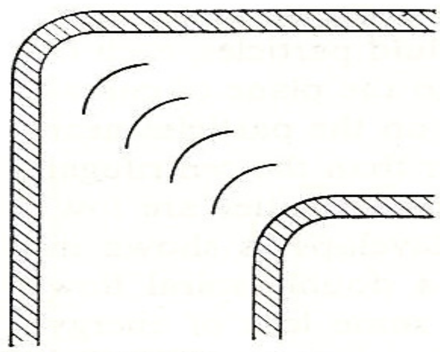


Figure 8.22 Vaned elbow.

Most of the loss of head in a sharp bend may be eliminated by the use of a vaned elbow, such as is shown in Fig. 8.22. The vanes tend to impede the formation of the secondary flows that would otherwise occur.

The head loss produced by a bend ($h_b = k_b V^2/2g$) in excess of the loss for an equal length of straight pipe is greatly dependent upon the ratio of the radius of curvature r to the diameter of the pipe D, and combinations of different pipe bends placed close together cannot be treated by adding up the losses of each one considered separately. The total loss depends not only upon the spacing between the bends, but also upon the relations of the directions of the bends and the planes in which they are located. Bend loss is not proportional to the angle of the bend; for 22.5 and 45° bends the losses are respectively about 40 and 80 per cent of the loss in a 90° bend. Typically for a 90° bend, k_b varies for a smooth pipe from 0.15 for $r/D = 2$ to 0.10 for $r/D = 10$, and for a pipe with $e/D = 0.0020$, k_b varies from about 0.30 to 0.20 for r/D values of 2 and 10 respectively. Information on values of k_b is available in the literature.[1]

[1] R. J. S. Pigott, Pressure Losses in Tubing, Pipe, and Fittings, *Trans. ASME*, vol. 72, p. 679, July, 1950. See also: E. F. Brater and H. W. King, "Handbook of Applied Hydraulics," 6th ed., McGraw-Hill Book Co., New York, N.Y., 1976.

8.22 SOLUTION OF PIPE-FLOW PROBLEMS

We have examined the fundamental fluid mechanics associated with the frictional loss of energy in pipe flow. While the interest of the scientist extends very little beyond this, it is the task of the engineer to apply these fundamentals to various types of practical problems. Pipe flow problems may be solved using the Hazen–Williams equation (8.45), the Manning equation (8.46) or the Darcy–Weisbach equation (8.13). The latter is to be preferred as it will provide greater accuracy since its application utilizes the basic parameters that influence pipe friction, namely, Reynolds number **R** and relative roughness e/D. To get good results with the Hazen–Williams and Manning equations the user must select proper values for C_{HW} and n respectively. This is more difficult than estimating the e/D ratio for a pipe as required by the Darcy–Weisbach equation. An advantage of the Manning equation is that all types of pipe flow problems can be solved directly by using it, while certain types of problems must be solved by trial and error when using the Darcy–Weisbach equation as discussed in Sec. 8.12. The Hazen–Williams equation is not well suited for the solution of all problems where minor losses must be considered because, upon rearranging Eq. (8.45), we find that in the Hazen–Williams equation the head loss due to pipe friction is proportional to $V^{1.85}$, while minor losses are expressed as being proportional to V^2.

In the typical direct-solution problem using the Darcy–Weisbach equation the head loss is determined for the given flow rate and pipeline and minor loss characteristics. The indirect-solution problems (trial and error) are of two principal types: (1) given the pipeline and minor loss characteristics and the head loss, find the flow rate and (2) given the flow rate and the energy gradient, find the required pipe diameter. The feature of these problems is the variation of f with Reynolds number. The usual procedure (see Illustrative Example 8.4) is to assume a reasonable value of f by referring to Fig. 8.11. This will then lead, through the pipe-friction and energy equations, to a *computed* velocity and Reynolds number. This determines a more accurate value of f, and it will generally be necessary to repeat the solution for new values of V and Q. As f varies little within a small range of **R**, a third trial will rarely be necessary.

The following example illustrates the method of solution for flow through a pipeline of uniform diameter.

Illustrative Example 8.5 Referring to Fig. 8.23, find the flow rate through a new 25 cm-diameter cast-iron pipe of length 1,500 m, with $\Delta z = 80$ m. Consider the entrance to be sharp-cornered nonprojecting.

From Fig. 8.12, $e/D \approx 0.001$. Referring to Fig. 8.11, assume $f = 0.020$. From Sec. 8.16 we choose a value of $k_e = 0.5$ for the loss at entrance. Then, writing the energy equation between the water surface and the free jet,

$$80 + 0 + 0 = 0 + 0 + \frac{V_2^2}{2g} + \left(0.5 + 0.02 \times \frac{1500}{0.25}\right)\frac{V_2^2}{2g}$$

This gives $V_2^2/2g = 0.66$ m and $V_2 = 3.6$ m/s. We may now confirm the trial value of f by returning to Fig. 8.11, with $1.3DV = 1.3 \times 25 \times 3.6 = 117$ and $e/D = 0.001$. Again, the chart shows $f = 0.020$, so no repeat solution is required. The flow is $Q = A_2V_2 = \pi/4(0.25)^2 \times 3.6 = 0.18$ m³/s.

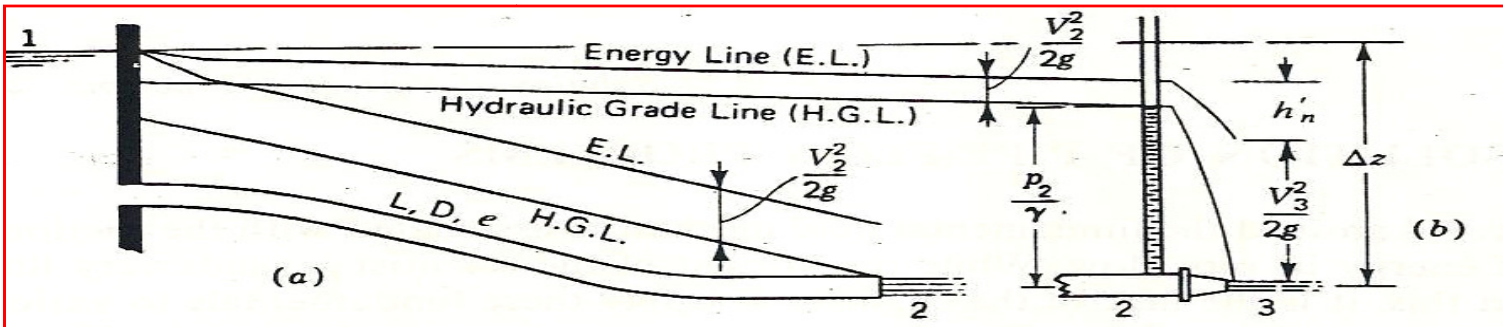


Figure 8.23 Discharge from a reservoir. (*a*) Free discharge. (*b*) With nozzle. As L/D gets larger the E.L. and H.G.L. approach one another.

In the foregoing example it may be seen that with this length of pipe it would have made very little difference if the entrance loss and also the velocity head at discharge had been neglected altogether. It is generally conceded that, for pipes of length greater than 1,000 diameters, the error incurred by neglecting minor losses is less than that inherent in selecting a value of f. In applying this rule one must of course use common sense and recall that a valve, for example, is a minor loss only when it is wide open. Partially closed, it may be the most important loss in the system.

If the pipe discharged into a fluid that was at a pressure other than atmospheric, the proper value of p_2/γ would have to be used in the energy equation.

Another example of flow from a reservoir is that of a penstock leading to an impulse turbine. In this case the pipe does not discharge freely but ends in a nozzle (Fig. 8.23b), which has a known or assumed loss coefficient. The head loss in the nozzle, h'_n, is associated with the high issuing velocity head and is therefore not a minor loss. The procedure is to employ the equation of continuity to place all losses in terms of the velocity head in the pipe. This is the logical choice for the "common unknown" because the trial-and-error solution will again be built around the pipe friction loss rather than the nozzle loss.

Illustrative Example 8.6 In Fig. 8.23 suppose that the pipeline of the preceding example is now fitted with a nozzle at the end which discharges a jet 6.5 cm in diameter and which has a loss coefficient of 0.11. Find the flow rate.

Let point 2 now refer to the pipe at the base of the nozzle and point 3 be in the jet. The head loss in the nozzle is $0.11V_3^2/2g$. Writing the energy equation between 1 and 3, neglecting entrance loss,

$$80 + 0 + 0 = 0 + 0 + \frac{V_3^2}{2g} + \frac{1500}{0.25} f \frac{V_2^2}{2g} + 0.11\frac{V_3^2}{2g}$$

By continuity equation, $V_3^2/2g = (25/6.5)^4 V_2^2/2g = 219V_2^2/2g$. Thus $80 = (1.11 \times 219 + 6000f)\dfrac{V_2^2}{2g}$

A trial value of f is selected. Let $f = 0.02$ for the first assumption. $\dfrac{V_2^2}{2g} = \dfrac{80}{363} = 0.22$ m

Then $80 = (243 + 120)V_2^2/2g$, from which

and $V_2 = 2.08$ m/s. With $1.3DV = 1.3 \times 25 \times 2.08 = 67.6$ and $e/D = 0.001$, Fig. 8.11 shows $f = 0.02$. In this case the first solution may be considered sufficiently accurate, but in general the value of f determined from the chart may be materially different from that assumed, and a second trial may be necessary. The rate of discharge is $Q = A_2V_2 = \pi/4(0.25)^2 \times 2.08 = 0.1$ m³/s

$$V_3 = \left(\frac{25}{6.5}\right)^2 V_2 = 14.8 \times 2.08 = 30.8 \text{ m/s}$$

As additional information, $H_2 = p_2/\gamma + V_2^2/2g = 80 - 0.02 \times 6{,}000 \times 0.22 = 53.6$ m, and the pressure head $p_2/\gamma = 53.6 - 0.22 = 53.38$ m.

This example shows that the addition of the nozzle has reduced the discharge from 0.18 to 0.1 m³/s but has increased the jet velocity from 3.6 to 30.8 m/s. The head loss due to pipe friction is 26.4 m and the head loss through the nozzle is 5.32 m. (The head loss at entrance which was neglected in the calculations is approximately 0.1 m.)

We may change Illustrative Example 8.5 into a type-2 indirect-solution problem by specifying the rate of discharge and finding the required diameter. Although this type of problem can be attacked in exactly the same way as the foregoing, the solution is facilitated by a slightly different procedure if the length is so great that the minor losses are negligible. From the continuity equation, $V = Q/A = 4Q/\pi D^2$. Substituting this expression for V in the pipe-friction equation,

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

and rearranging, we obtain

$$\frac{D^5}{f} = \frac{8LQ^2}{\pi^2 g h_L} = \text{constant} \qquad (8.52)$$

A value of f may be assumed more or less arbitrarily and an approximate value of the pipe diameter computed by this equation. This determines the velocity, Reynolds number, and relative roughness. A new value of f is determined with the aid of Fig. 8.11, and the computation may be repeated if necessary. In general, the diameter so obtained will not be a standard pipe size, and the size selected will usually be the next largest commercially available size. In planning for the future it must be recalled that scale deposits will increase the roughness and reduce the cross-sectional area. For pipes in water service, the absolute roughness e of old pipes (twenty years and more) may increase over that of new pipes by threefold for concrete or cement-lined steel, up to twentyfold for cast iron, and even to fortyfold for tuberculated wrought-iron and steel pipe. Equation (8.52) shows that for a constant value of f, Q varies as $D^{5/2}$. Hence for the case where minor losses are negligible and f is constant, to achieve a 100 percent increase in flow, the diameter need be increased only 32 percent. This amounts to a 74 percent increase in cross-sectional area.

If the minor losses and the velocity head in the pipe are not negligible in comparison with the pipe friction, the problem may be handled by expressing such losses in equivalent lengths of pipe, and the solution reduces to the case just described. This approach can be used if the pipe behaves as wholly rough (Sec. 8.11) in which case f depends only on e/D and is independent of **R**. The length equivalence of a minor loss is obtained by equating $(fL/D)V^2/2g$ to $(k)V^2/2g$. From this one obtains the equivalent length of pipe as $L_e = kD/f$ where k is the minor loss coefficient.

8.23 PIPELINE WITH PUMP OR TURBINE

If a pump lifts a fluid from one reservoir to another, as in Fig. 8.24, not only does it do work in lifting the fluid the height Δz, but also it has to overcome the friction loss in the suction and discharge piping. This friction head is equivalent to some added lift, so that the effect is the same as if the pump lifted the fluid a height $\Delta z + \sum h_L$. Hence the power delivered to the liquid by the pump is $\gamma Q(\Delta z + \sum h_L)$. The power required to run the pump is greater than this, depending on the efficiency of the pump. The total pumping head h_p for this case is

$$h_p = \Delta z + \sum h_L \tag{8.53}$$

If the pump discharges a stream through a nozzle, as shown in Fig. 8.25, not only has the liquid been lifted a height Δz, but also it has received a kinetic energy head of $V_2^2/2g$, where V_2 is the velocity of the jet. Thus the total pumping head is now

$$h_p = \Delta z + \frac{V_2^2}{2g} + \sum h_L \tag{8.54}$$

In any case the total pumping head may be determined by writing the energy equation between any point upstream from the pump and any other point downstream, as in Eq. (4.14). For example, if the upstream reservoir were at a higher elevation than the downstream one, then the Δz's in the two foregoing equations would have negative signs.

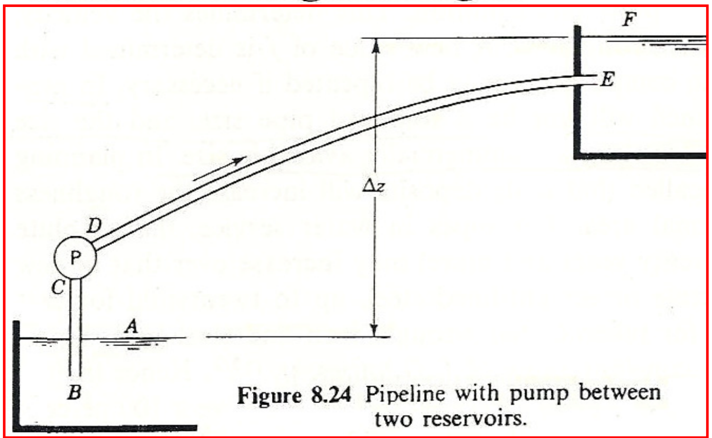


Figure 8.24 Pipeline with pump between two reservoirs.

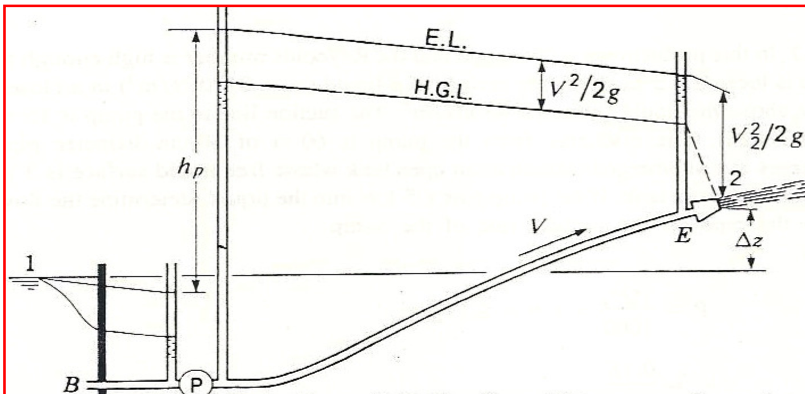


Figure 8.25 Pipeline with pump and nozzle.

The machine that is employed for converting the energy of flow into mechanical work is called a *turbine*. In flowing from the upper tank in Fig. 8.26 to the lower, the fluid loses potential energy head equivalent to Δz. This energy is expended in two ways, part of it in hydraulic friction in the pipe and the remainder in the turbine. Of that which is delivered to the turbine, a portion is lost in hydraulic friction and the rest is converted into mechanical work.

The power delivered to the turbine is decreased by the friction loss in the pipeline, and its value is given by $\gamma Q(\Delta z - \sum h_L)$. The power delivered by the machine is less than this, depending upon both the hydraulic and mechanical losses of the turbine. The head under which the turbine operates is

$$h_t = \Delta z - \sum h_L \tag{8.55}$$

where $\sum h_L$ is the loss of head in the supply pipe and does not include the head loss in the draft tube (DE in Fig. 8.26), since the draft tube is considered an integral part of the turbine. The draft tube has a gradually increasing cross-sectional area which results in a reduced velocity at discharge. This enhances the

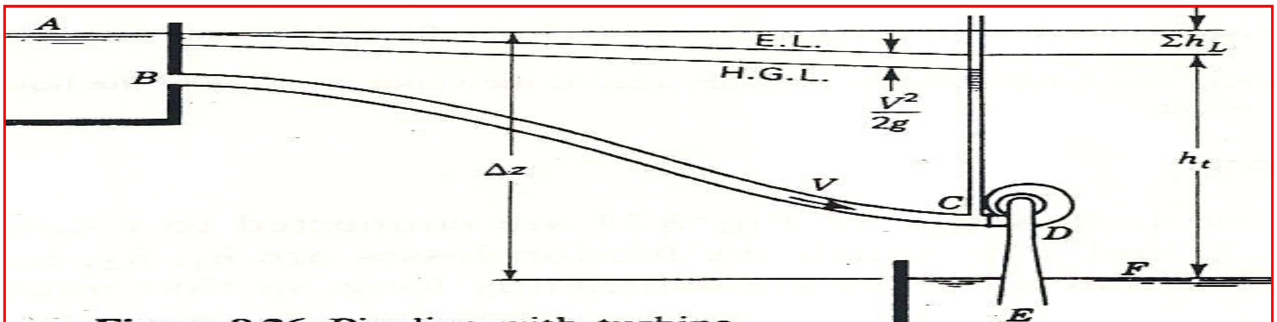


Figure 8.26 Pipeline with turbine.

efficiency of the turbine because of the reduced head loss at discharge (Sec. 16.6). It should be noted that the h_t of Eq. (8.55) represents the energy head removed from the fluid by the turbine; this, of course, is identical to the energy head transferred to the turbine from the fluid.

Illustrative Example 8.7 In this problem we will assume that the Reynolds number is high enough to assure turbulent flow. A pump is located 4.5 m above the surface of a liquid ($\gamma = 8170$ N/m³) in a closed tank. The pressure in the space above the liquid surface is 35 kN/m². The suction line to the pump is 15 m of 15 cm-diameter pipe ($f = 0.025$). The discharge from the pump is 60 m of 20 cm-diameter pipe ($f = 0.030$). This pipe discharges in a submerged fashion to an open tank whose free liquid surface is 3 m lower than the liquid surface in the pressure tank. If the pump puts 1.5 kW into the liquid, determine the flow rate and find the pressure in the pipe on the suction side of the pump.

From Eq. (4.16), $\quad P = \frac{\gamma Q h_p}{1000} = 1.5 = 8.17 Q h_p \quad$ Thus $\quad h_p = \frac{0.18}{Q}$

Writing the energy equation from one liquid surface to the other,

$$\frac{35000}{8170} - 0.5\frac{V_{15}^2}{2g} - 0.025\left(\frac{15}{0.15}\right)\frac{V_{15}^2}{2g} + h_p - 0.030\left(\frac{60}{0.20}\right)\frac{V_{20}^2}{2g} = -3 + \frac{V_{20}^2}{2g}$$

Substituting $V_{15} = Q/0.01767$ and $V_{20} = Q/0.03142$ this reduces to

$$4.28 - 81.62Q^2 - 408.1Q^2 + \frac{0.18}{Q} - 464.66Q^2 = -3 + 51.63Q^2 \quad \text{or} \quad 1006Q^3 - 7.28Q - 0.18 = 0$$

By trial, $\quad Q = 0.0955$ m³/s

To obtain the pressure at the suction side of the pump, pump قبل

$$\frac{35000}{8170} - 0.5\frac{V_{15}^2}{2g} - 0.025\left(\frac{15}{0.15}\right)\frac{V_{15}^2}{2g} = 4.5 + \frac{p}{\gamma} + \frac{V_{15}^2}{2g} \quad \text{where} \quad V_{15} = \frac{0.0955}{0.01769} = 5.4 \text{ m/s}$$

$$\therefore \quad 4.28 - 0.74 - 3.72 = 4.5 + p/\gamma + 1.49 \quad \text{from which} \quad \frac{p}{\gamma} = -6.17 \text{ m}$$

or $p_2 = -6.17(8170) = -50.4$ kN/m² which is equivalent to $(50.4/101.32)(760) = 378$ mm of mercury vacuum.

In this type of problem one should check the absolute pressure against the vapor pressure of the liquid to see that vaporization does not occur.

8.24 BRANCHING PIPES

Suppose that three reservoirs *A*, *B*, and *C* of Fig. 8.27 are connected to a common junction *J* by pipes 1, 2, and 3, in which the friction losses are h_1, h_2, and h_3, respectively. It is supposed that all pipes are sufficiently long, so that minor losses and velocity heads may be neglected. Actually, any one of the pipes may be considered leading to or from some destination other than the reservoir shown by simply replacing the reservoir with a piezometer tube in which the water level is the same as that of the reservoir surface. The continuity and energy equations require that the flow entering the junction equal the flow leaving it and that the pressure head at *J* (which may be represented schematically by the open piezometer tube shown, with water at elevation *P*) be common to all pipes. That is, for the condition shown: 1. $Q_1 = Q_2 + Q_3$. 2. Elevation *P* is common to all.
If *P* is below the surface of *B*, then the flow will be out of *B* and $Q_1 + Q_2 = Q_3$. The diagram suggests several problems, three of which will be discussed below:

1. Given all pipe lengths and diameters, the surface elevations of two reservoirs, and the flow to or from one of these two, find the surface elevation of the third reservoir. This is a direct-solution problem. Suppose that Q_1 and the elevations of *A* and *B* are given. The head loss h_1 is determined directly by the pipe-friction equation, using Fig. 8.11 to determine the proper value of *f*. This fixes *P* and h_2 was given. The flow in pipe 2 may then be determined, assuming a reasonable value of *f* and adjusting it if necessary. Condition 1 (continuity at the junction) then determines Q_3, which in turn determines h_3 and the surface elevation of *C*.
2. Given all pipe lengths and diameters, the elevations of water surfaces of two reservoirs, and the flow to or from the third, find the elevation of the surface in the third reservoir. Suppose Q_2 and the surface elevations of *A* and *C* are given. Then the quantities $Q_1 - Q_3$ and $h_1 + h_3$ are known. These relations are solved simultaneously for their component parts in one of two ways: (*a*) by assuming successive distributions of the flows Q_1 and Q_2 satisfying the first relation, until a distribution is found which also satisfies the head-loss relation; (*b*) by assuming successive elevations of the piezometer level *P*, which is

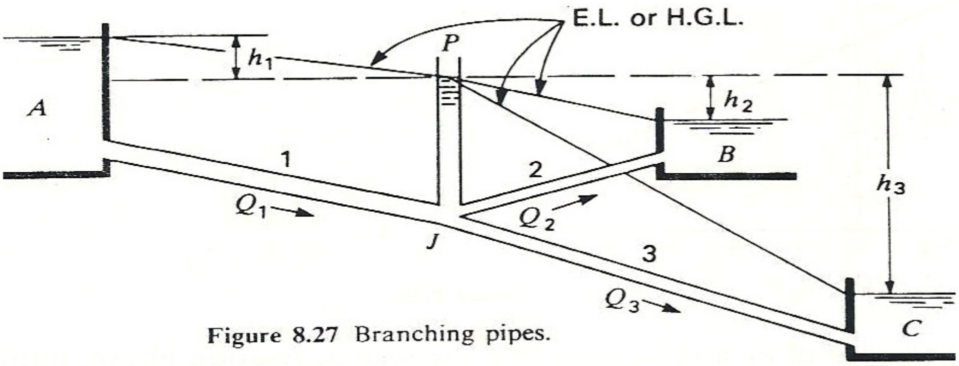


Figure 8.27 Branching pipes.

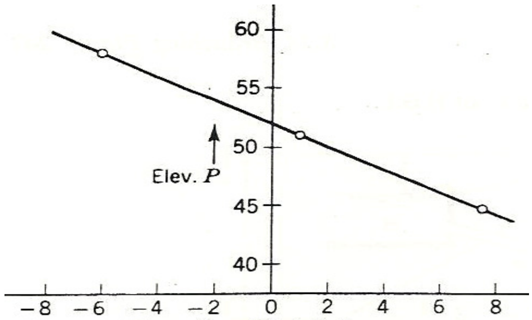


Figure 8.28 $Q_1 - (Q_2 + Q_3)$

to say, distributions of h_1 and h_3 satisfying the second relation above, until a level is found which also satisfies the discharge relation. With P known and h_2 determined by the given discharge Q_2, the elevation of B is easily obtained.

3. Given all pipe lengths and diameters and the elevations of all three reservoirs, find the flow in each pipe. This is the classic *three-reservoir problem*, and it differs from the foregoing cases in that it is not immediately evident whether the flow is *into* or *out of* reservoir B. This direction is readily determined by first assuming no flow in pipe 2; that is, the piezometer level P is assumed at the elevation of the surface of B. The head losses h_1 and h_3 then determine the flows Q_1 and Q_3, and depending on whether $Q_1 > Q_3$ or $Q_1 < Q_3$, the condition of continuity is determined as $Q_1 = Q_2 + Q_3$ or $Q_1 + Q_2 = Q_3$, respectively. From this point the solution proceeds as in (*b*) of case 2 above. The piezometer level is moved up or down by trial until the resulting flow distribution satisfies the continuity relation. In reaching the final adjusted level it is helpful to make a small plot such as is shown in Fig. 8.28 for the case where $Q_1 = Q_2 + Q_3$. Two or three points, with one fairly close to the axis, determine a curve which intersects the vertical axis at the equilibrium level of P, that is, for the condition $Q_1 - (Q_2 + Q_3) = 0$.

8.25 PIPES IN SERIES[1]

The discussion in Sec. 8.22 was restricted to the case of a single pipe. If the pipe is made up of sections of different diameters, as shown in Fig. 8.29, the continuity and energy equations establish the following two simple relations which must be satisfied:

$$Q = Q_1 = Q_2 = Q_3 = \cdots \qquad (8.56)$$

$$h_L = h_{L1} + h_{L2} + h_{L3} + \cdots \qquad (8.57)$$

[1] Once again it should be mentioned that either the Hazen–Williams or Manning equation can be used to solve pipe-flow problems, though the Darcy–Weisbach approach is best.

In case Eqs. (8.45) or (8.46) are being used, the equivalent length is established by the relation $S = h_L/L$, or $L_e = L(S/S_e)$, where the values of the energy gradient are obtained for any assumed rate of discharge. Minor losses can also be handled using the equivalent-length method.

Illustrative Example 8.8 Suppose in Fig. 8.29 the pipes 1, 2, and 3 are 300 m of 30-cm-diameter, 150 m of 20-cm-diameter, and 250 m of 25-cm-diameter, respectively, of new cast iron and are conveying 15°C water. If $h = 10$ m, find the rate of flow from A to B.

(*a*) BY THE EQUIVALENT-VELOCITY-HEAD METHOD. For cast-iron pipe $e = 0.25$ mm (Table 8.1); hence the corresponding values for e/D are: 0.00083, 0.00125, and 0.0010, and from Fig. 8.11 we will assume $f_1 = 0.019$, $f_2 = 0.021$, and $f_3 = 0.020$. Then,

$$h = 10 = 0.019\left(\frac{300}{0.3}\right)\frac{V_1^2}{2g} + 0.021\left(\frac{150}{0.2}\right)\frac{V_2^2}{2g} + 0.020\left(\frac{250}{0.25}\right)\frac{V_3^2}{2g}$$

From continuity $\quad \dfrac{V_2^2}{2g} = \dfrac{V_1^2}{2g}\left(\dfrac{D_1}{D_2}\right)^4 = \dfrac{V_1^2}{2g}\left(\dfrac{30}{20}\right)^4 = 5.06\,\dfrac{V_1^2}{2g} \qquad$ Similarly $\quad \dfrac{V_3^2}{2g} = 2.07\,\dfrac{V_1^2}{2g}$

and thus $\quad 10 = \dfrac{V_1^2}{2g}\left(0.019\,\dfrac{1{,}000}{1} + 0.021\,\dfrac{750}{1}\,5.06 + 0.020\,\dfrac{1{,}000}{1}\,2.07\right) \qquad$ from which $\quad \dfrac{V_1^2}{2g} = 0.071$ m

Hence $\quad V_1 = \sqrt{2(9.81\text{ m/s}^2)(0.072\text{ m})} = 1.18$ m/s

The corresponding values of $\mathbf{R}$ are 0.31×10^6, 0.47×10^6, and 0.37×10^6; the corresponding friction factors are only slightly different from those originally assumed since the flow is at Reynolds numbers very close to those at which the pipes behave as rough pipes.

Hence $\quad Q = A_1V_1 = \dfrac{\pi}{4}(0.30)^2 1.18 = 0.083\text{ m}^3/\text{s}$

Greater accuracy would have been obtained if the friction factors had been adjusted to match the pipe-friction chart more closely and if minor losses had been included. In that case, $Q = 0.081\text{ m}^3/\text{s}$.

(*b*) BY THE EQUIVALENT-LENGTH METHOD. Choose the 30-cm pipe as the standard. Using the above values of f in Eq. (8.59)

Pipe 2: $\quad L_e = 150\left(\dfrac{0.021}{0.019}\right)\left(\dfrac{30}{20}\right)^5 = 1{,}260$ m of 30-cm pipe

Pipe 3: $\quad L_e = 250\left(\dfrac{0.020}{0.019}\right)\left(\dfrac{30}{25}\right)^5 = 650$ m of 30-cm pipe

Add pipe 1 = 300 m of 30-cm pipe $\qquad$ Total $L_e = 2{,}210$ m of 30-cm pipe

Thus $\quad h = 10 = 0.019\,\dfrac{2{,}210}{0.30}\,\dfrac{V_1^2}{2g} \qquad \dfrac{V_1^2}{2g} = 0.071$ m $\quad V_1 = 1.18$ m/s and $Q = 0.083\text{ m}^3/\text{s}$ as above.

8.26 PIPES IN PARALLEL

In the case of flow through two or more parallel pipes, as in Fig. 8.30, the continuity and energy equations establish the following relations which must be satisfied:

$$Q = Q_1 + Q_2 + Q_3 \tag{8.60}$$

and

$$h_L = h_{L1} = h_{L2} = h_{L3} \tag{8.61}$$

as the pressures at A and B are common to all pipes. If the head loss is given, the total discharge may be computed directly by adding the contributions from the various pipes, as in Eq. (8.60).

If the total flow is given and the head loss and distribution of flow among the pipes are required, an approximate solution may be obtained by assuming a reasonable value of h_L and computing the resulting individual flows and the *percentage* distribution of flow. This percentage distribution will not change greatly with the magnitude of the flow and may then be applied to find the actual distribution of the total discharge. The accuracy of the solution may be checked by comparing the computed head losses in the separate pipes. They should be the same. If they are not the same, the assumed values of f can be corrected to match the Moody diagram (Fig. 8.11). A more accurate procedure is to write Eq. (8.61) for the flow in each pipe in terms of the dimensions applying to it. This may be accomplished by observing that the loss of head in any pipe is

$$h_L = \left(f\frac{L}{D} + \sum k\right)\frac{V^2}{2g}$$

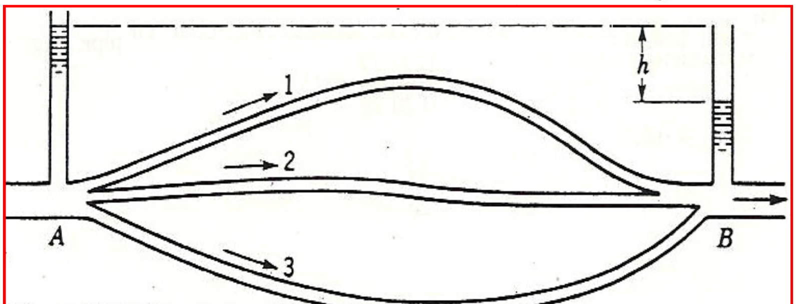
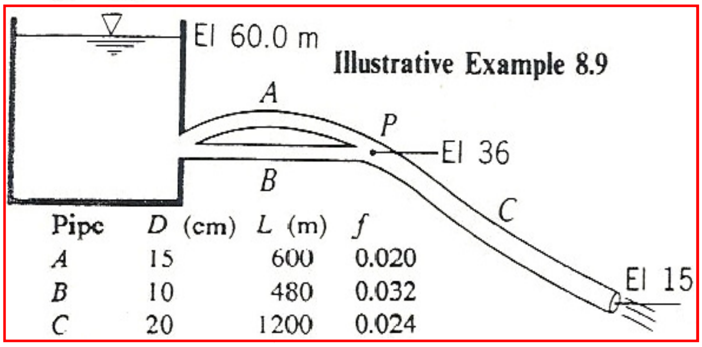


Figure 8.30 Pipes in parallel.



Pipe	D (cm)	L (m)	f
A	15	600	0.020
B	10	480	0.032
C	20	1200	0.024

where $\sum k$ is the sum of the minor-loss coefficients, which may usually be neglected if the pipe is longer than 1,000 diameters. Solving for V and then Q, the following is obtained for pipe 1:

$$Q_1 = A_1V_1 = A_1\sqrt{\frac{2gh_L}{f_1(L_1/D_1) + \sum k}} = C_1\sqrt{h_L} \tag{8.62}$$

where C_1 is constant for the given pipe, except for the change in f with Reynolds number. The flows in the other pipes may be similarly expressed, using reasonable values of f from Fig. 8.11. Finally, Eq. (8.57) becomes

$$Q = C_1\sqrt{h_L} + C_2\sqrt{h_L} + C_3\sqrt{h_L} = \sqrt{h_L}(C_1 + C_2 + C_3)$$

This enables a first determination of h_L and the distribution of flows and velocities in the pipes. Adjustments in the values of f may be made next, if indicated, and finally a corrected determination of h_L and the distribution of flows.

It is instructive to compare the solution methods for pipes in parallel with those for pipes in series. The role of the head loss in one case becomes that of the discharge rate in the other, and vice versa. The student is already familiar with this situation from the elementary theory of dc circuits. The flow corresponds to the electrical current, the head loss to the voltage drop, and the frictional resistance to the ohmic resistance. The outstanding deficiency in this analogy occurs in the variation of potential drop with flow, which is with the first power in the electrical case ($E = IR$) and with the second power in the hydraulic case ($h_L \propto V^2 \propto Q^2$) for fully developed turbulent flow.

Illustrative Example 8.9 Three pipes A, B, and C are interconnected as shown. The pipe characteristics are as follows:

Find the rate at which water will flow in each pipe. Find also the pressure at point P. All pipe lengths are much greater than 1,000 diameters, therefore minor losses may be neglected.

Energy Eq: $60 - 0.020\dfrac{600}{0.15}\dfrac{V_A^2}{2g} - 0.024\dfrac{1200}{0.20}\dfrac{V_C^2}{2g} = 15$ i.e. $45 = 80\dfrac{V_A^2}{2g} + 144\dfrac{V_C^2}{2g}$

Continuity: $15^2V_A + 10^2V_B = 20^2V_C$ i.e. $225V_A + 100V_B = 400V_C$

Also, $h_{L_A} = h_{L_B} = 0.020\dfrac{600}{0.15}\dfrac{V_A^2}{2g} = 0.032\dfrac{480}{0.10}\dfrac{V_B^2}{2g}$ i.e. $80V_A^2 = 153.6V_B^2$, $V_B = 0.722V_A$

Substituting into continuity, $225V_A + 100(0.722V_A) = 400V_C$ $297.2V_A = 400V_C$ $V_A = 1.346V_C$

Substituting into the energy equation, $45 = 80\dfrac{(1.346V_C)^2}{2g} + 144\dfrac{V_C^2}{2g} = 288.9\dfrac{V_C^2}{2g}$

$V_C^2 = 19.62(45/288.9) = 3.056$ $V_C = 1.75$ m/s $Q_C = A_CV_C = (0.03142)1.75 = 0.055$ m^3/s

$V_A = 1.346V_C = 2.35$ m/s $Q_A = (0.01767)2.35 = 0.042$ m^3/s

Continuity: $225(2.35)+100V_B=400(1.75)$ $V_B=1.70$ m/s $Q_B=A_BV_B=(0.00785)1.7=0.013$ m^3/s

As a check, note that $Q_A+Q_B=Q_C$

To find the pressure at P $\quad 60-80\frac{V_A^2}{2g}=36+p_P/\gamma \quad p_P/\gamma=24-80\frac{(2.35)^2}{2g}=1.48$ m

Check: $\quad 36+p_P/\gamma-144\frac{V_C^2}{2g}=15 \quad p_P/\gamma=144\frac{(1.75)^2}{2g}-21=1.48$ m

So $p_P/\gamma=1.48$ m and $p_P=1.48\times 9810=14.52$ kN/m^2

In this example it was assumed that the values of f for each pipe were known. Actually f depends on **R** (Fig. 8.11). Usually the absolute roughness e of each pipe is known or assumed and an accurate solution is achieved through trial and error until the f's and **R**'s for each pipe agree with the Moody diagram (Fig. 8.11).

8.27 PIPE NETWORKS

An extension of pipes in parallel is a case frequently encountered in municipal distribution systems, in which the pipes are interconnected so that the flow to a

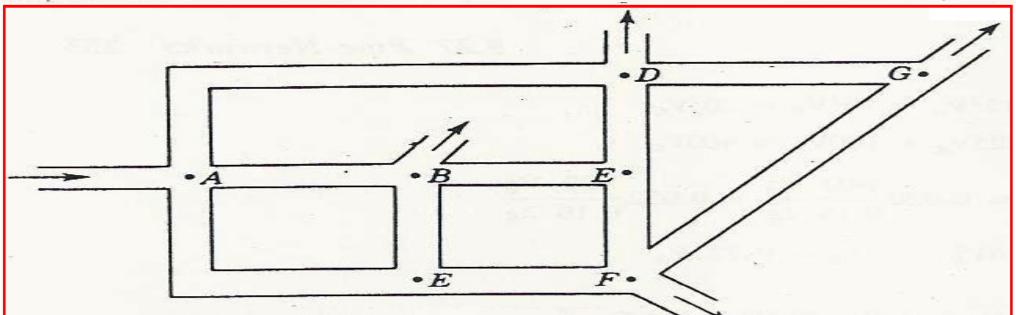


Figure 8.31 Pipe network.

given outlet may come by several different paths, as shown in Fig. 8.31. Indeed, it is frequently impossible to tell by inspection which way the flow travels, as in pipe BE. Nevertheless, the flow in any network, however complicated, must satisfy the basic relations of continuity and energy as follows:

1. The flow into any junction must equal the flow out of it.
2. The flow in each pipe must satisfy the pipe-friction laws for flow in a single pipe.
3. The algebraic sum of the head losses around any closed loop must be zero.

Pipe networks are generally too complicated to solve analytically, as was possible in the simpler cases of parallel pipes (Sec. 8.26). A practical procedure is the method of successive approximations, introduced by Cross.[1] It consists of the following elements, in order:

1. By careful inspection assume the most reasonable distribution of flows that satisfies conditions 1.
2. Write condition 2 for each pipe in the form
$$h_L=KQ^n \tag{8.63}$$
where K is a constant for each pipe. For example, the standard pipe-friction equation in the form of Eq. (8.62) would yield $K=1/C^2$ and $n=2$ for constant f. The empirical formulas (8.45) and (8.46) are seen to be readily reducible to the desired form. Minor losses within any loop may be included, but minor losses at the junction points are neglected.
3. To investigate condition 3, compute the algebraic sum of the head losses around each elementary loop, $\sum h_L=\sum KQ^n$. Consider losses from clockwise flows as positive, counterclockwise negative. Only by good luck will these add to zero on the first trial.

[1] Hardy Cross, Analysis of Flow in Networks of Conduits or Conductors, *Univ. Ill. Eng. Expt. Sta. Bull.* 286, 1936.

4. Adjust the flow in each loop by a correction, ΔQ, to balance the head in that loop and give $\sum KQ^n=0$. The heart of this method lies in the determination of ΔQ. For any pipe we may write $\qquad Q=Q_0+\Delta Q$

where Q is the correct discharge and Q_0 is the assumed discharge. Then, for each pipe, $\quad h_L=KQ^n=K(Q_0+\Delta Q)^n=K(Q_0^n+nQ_0^{n-1}\,\Delta Q+\cdots)$

If ΔQ is small compared with Q_0, the terms of the series after the second one may be neglected. Now, for a circuit, with ΔQ the same for all pipes,
$$\sum h_L=\sum KQ^n=\sum KQ_0^n+\Delta Q\sum KnQ_0^{n-1}=0$$

As the corrections of head loss in all pipes must be summed *arithmetically*, we may solve this equation for ΔQ,

$$\Delta Q = \frac{-\sum KQ_0^n}{\sum |KnQ_0^{n-1}|} = \frac{-\sum h_L}{n \sum |h_L/Q_0|} \tag{8.64}$$

as, from Eq. (8.63), $h_L/Q = KQ^{n-1}$. It must be emphasized again that the numerator of Eq. (8.64) is to be summed algebraically, with due account of sign, while the denominator is summed arithmetically. The negative sign in Eq. (8.64) indicates that when there is an excess of head loss around a loop in the clockwise direction, the ΔQ must be subtracted from clockwise Q_0's and added to counterclockwise ones. The reverse is true if there is a deficiency of head loss around a loop in the clockwise direction.

5. After each circuit is given a first correction, the losses will still not balance because of the interaction of one circuit upon another (pipes which are common to two circuits receive two independent corrections, one for each circuit). The procedure is repeated, arriving at a second correction, and so on, until the corrections become negligible.

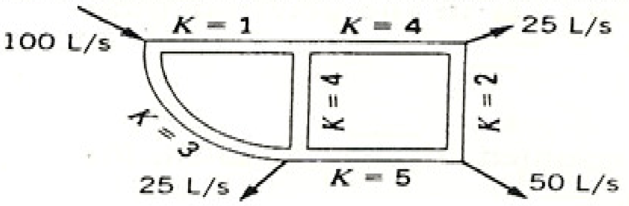
Either form of Eq. (8.64) may be used to find ΔQ. As values of K appear in both numerator and denominator of the first form, values proportional to the actual K may be used to find the distribution. The second form will be found most convenient for use with pipe-friction diagrams for water pipes.

An attractive feature of the approximation method is that errors in computation have the same effect as errors in judgment and will eventually be corrected by the process.

The pipe-network problem lends itself well to solution by use of a digital computer.[1] Programming takes time and care, but once set up, there is great flexibility and many man-hours of labor can be saved.

[1] Lyle N. Hoag and Gerald Weinberg, Pipeline Network Analysis by Electronic Digital Computer, *J. Am. Water Works Assoc.*, vol. 49. pp. 517–529, 1957.

Illustrative Example 8.10 If the flows into and out of a two-loop pipe system are as shown, determine the flow in each pipe. The K values for each pipe were calculated from the pipe and minor loss characteristics and from an assumed value of f.



As a first step, assume flow in each pipe such that continuity is satisfied at all junctions. Calculate ΔQ for each loop, make corrections to the assumed Q's and repeat several times until the ΔQ's are quite small. As a final step the values of f for each pipe should be checked against the Moody diagram and modified, if necessary.

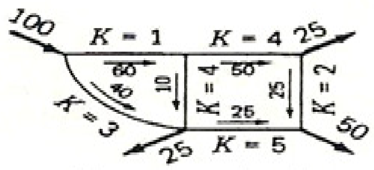
Left loop

ΣKQ_0^n	$\Sigma\lvert KnQ_0^{n-1}\rvert$
$1 \times 60^2 = 3{,}600$	$1 \times 2 \times 60 = 120$
$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$
4,000)	
$3 \times 40^2 = 4{,}800$)	$3 \times 2 \times 40 = 240$
800)	440

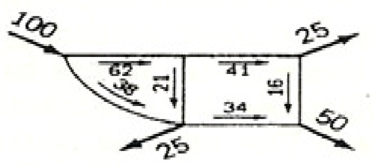
$$\Delta Q_1 = \frac{800}{440} \approx 2\ )$$

$1 \times 62^2 = 3{,}844$	$1 \times 2 \times 62 = 124$
$4 \times 21^2 = 1{,}764$	$4 \times 2 \times 21 = 168$
5,608)	
$3 \times 38^2 = 4{,}332$)	$3 \times 2 \times 38 = 228$
1,276)	520

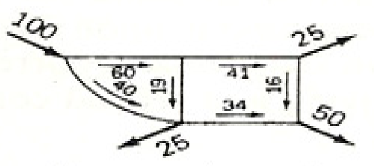
$$\Delta Q_2 = \frac{1{,}276}{520} \approx 2\ )$$



First approximation



After first correction



After second correction

Right loop

ΣKQ_0^n	$\Sigma\lvert KnQ_0^{n-1}\rvert$
$4 \times 50^2 = 10{,}000$	$4 \times 2 \times 50 = 400$
$2 \times 25^2 = 1{,}250$	$2 \times 2 \times 25 = 100$
11,250)	
$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$
$5 \times 25^2 = 3{,}125$	$5 \times 2 \times 25 = 250$
3,525)	830
7,725)	

$$\Delta Q_1 = \frac{7{,}725}{830} \approx 9\ )$$

$4 \times 41^2 = 6{,}724$	$4 \times 2 \times 41 = 328$
$2 \times 16^2 = 512$	$2 \times 2 \times 16 = 64$
7,236)	
$4 \times 21^2 = 1{,}764$	$4 \times 2 \times 21 = 168$
$5 \times 34^2 = 5{,}780$	$5 \times 2 \times 34 = 340$
7,544)	900
308)	

$$\Delta Q_2 = \frac{308}{900} \approx 0$$

Illustrative Example 8.10

Further corrections can be made if greater accuracy is desired.

PROBLEMS

8.1 An oil with a kinematic viscosity of 0.135 St flows through a pipe of diameter 15 cm. Below what velocity will the flow be laminar?

8.2 An oil with a kinematic viscosity of 0.0005 m^2/s flows through a 7.5 cm-diameter pipe with a velocity of 3 m/s. Is the flow laminar or turbulent?

8.3 Hydrogen at atmospheric pressure and a temperature of 10°C has a kinematic viscosity of 0.0001 m^2/s. Determine the maximum laminar flow rate in newtons per second in a 5 cm-diameter pipe. At this flow rate what is the average velocity?

8.4 Air at a pressure of approximately 1,500 kN/m^2, abs and a temperature of 100°C flows in a 1.5-cm-diameter tube. What is the maximum laminar flow rate? Express answer in liters per second, newtons per second, and kilograms per second. At this flow rate what is the average velocity?

8.5 What is the hydraulic radius of a rectangular air duct 15 by 35 cm?

8.6 What is the percentage difference between the hydraulic radii of a 20-cm-diameter and a 20-cm-square duct?

8.7 Two pipes, one circular and one square, have the same cross-sectional area. Which has the larger hydraulic radius, and by what percentage?

8.8 Steam with a specific weight of 40 N/m^3 flows with a velocity of 30 m/s through a circular pipe. The friction factor f was found to have a value of 0.016. What is the shearing stress at the wall?

8.9 Find the head loss per unit length when oil ($s = 0.9$) of viscosity 0.00065 m^2/s flows in a 7.5-cm-diameter pipe at a rate of 0.30 L/s.

8.10 Tests made on a certain 30 cm-diameter pipe showed that, when $V = 3$ m/s, $f = 0.015$. The fluid used was water at 15°C. Find the unit shear at the wall and at radii of 0, 0.2, 0.3, 0.5, 0.75 times the pipe radius.

8.11 If the oil of Prob. 8.2 weighs 9.11 kN/m^3, what will be the flow rate and head loss in a 900 m length of 10 cm-diameter pipe when the Reynolds number is 800?

8.12 With laminar flow in a circular pipe, at what distance from the centerline does the average velocity occur?

8.13 With laminar flow in a circular pipe, find the velocities at $0.1r$, $0.3r$, $0.5r$, $0.7r$, and $0.9r$. Plot the velocity profile.

8.14 Prove that the centerline velocity is twice the average velocity when laminar flow occurs in a circular pipe.

8.15 When laminar flow occurs in a two-dimensional passage, find the relation between the average and maximum velocities.

8.16 With laminar flow between two parallel, flat plates a small distance d apart, at what distance from the centerline will the velocity be equal to the mean velocity?

8.17 How much power is lost per meter of pipe length when oil with a viscosity of 0.20 $N\cdot s/m^2$ flows in a 20-cm-diameter pipe at 0.50 L/s? The oil has a density of 840 kg/m^3.

8.18 In Prob. 8.2 what will be the approximate distance from the pipe entrance to the first point at which the flow is established?

8.19 The absolute viscosity of water at 15°C is 0.001139 $N\cdot s/m^2$. (*a*) If at a distance of 7.5 cm from the center of the pipe of Prob. 8.10 the velocity profile gives a value for du/dy of 4.34 per second, find the viscous shear and the turbulent shear at that radius. (*b*) What is the value of the mixing length l, and what is the value of the ratio l/r_0?

8.20 Water at 15°C enters a pipe with a uniform velocity of 3 m/s. (*a*) What is the distance at which the transition occurs from a laminar to a turbulent boundary layer? (*b*) If the thickness of this initial laminar boundary layer is given by $4.91\sqrt{\nu x/U}$, what is the thickness reached by it at the point of transition?

8.21 Water in a pipe is at a temperature of 15°C. (*a*) If the mean velocity is 3.5 m/s, and the value of f is 0.015, what is the nominal thickness δ_l of the viscous sublayer? (*b*) What will be the thickness of the viscous sublayer if the velocity is increased to 5.8 m/s and f does not change?

8.22 For the data in Prob. 8.21(*a*), what is the distance from the wall to the assumed limit of the transition region where true turbulent flow begins?

8.23 Water at 40°C flows in a 20-cm-diameter pipe with $V = 5$ m/s and $e = 0.12$ mm. Head loss measurements indicate that $f = 0.022$. What is the thickness of the viscous sublayer? Is the pipe behaving as a wholly rough pipe?

10.7 NETWORKS OF PIPES

Interconnected pipes through which the flow to a given outlet may come from several circuits are called a *network of pipes*, in many ways analogous to flow through electric networks. Problems on these in general are complicated and require trial solutions in which the elementary circuits are balanced in turn until all conditions for the flow are satisfied.

The following conditions must be satisfied in a network of pipes:

1. The algebraic sum of the pressure drops around each circuit must be zero.
2. Flow into each junction must equal flow out of the junction.
3. The Darcy-Weisbach equation, or equivalent exponential friction formula, must be satisfied for each pipe; i.e., the proper relation between head loss and discharge must be maintained for each pipe.

The first condition states that the pressure drop between any two points in the circuit, for example, A and G (Fig. 10.12), must be the same whether through the pipe AG or through $AFEDG$. The second condition is the continuity equation.

Since it is impractical to solve network problems analytically, methods of successive approximations are utilized. The Hardy Cross method† is one in which flows are assumed for each pipe so that continuity is satisfied at every junction. A correction to the flow in each circuit is then computed in turn and applied to bring the circuits into closer balance.

Minor losses are included as equivalent lengths in each pipe. Exponential equations are commonly used, in the form $h_f = rQ^n$, where $r = RL/D^m$ in Eq. (10.1.1). The value of r is a constant in each pipeline (unless the Darcy-Weisbach equation is used) and is determined in advance of the loop-balancing procedure. The corrective term is obtained as follows.

† Hardy Cross, Analysis of Flow in Networks of Conduits or Conductors, *Univ. Ill. Bull.* 286, November 1936.

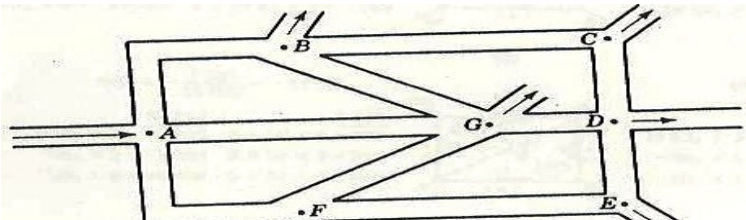


Figure 10.12
Pipe network.

For any pipe in which Q_0 is an assumed initial discharge

$$Q = Q_0 + \Delta Q \tag{10.7.1}$$

where Q is the correct discharge and ΔQ is the correction. Then for each pipe,

$$h_f = rQ^n = r(Q_0 + \Delta Q)^n = r(Q_0^n + nQ_0^{n-1}\,\Delta Q + \cdots)$$

If ΔQ is small compared with Q_0, all terms of the series after the second may be dropped. Now for a circuit,

$$\Sigma h_f = \Sigma rQ\,|Q|^{n-1} = \Sigma rQ_0\,|Q_0|^{n-1} + \Delta Q\,\Sigma rn\,|Q_0|^{n-1} = 0$$

in which ΔQ has been taken out of the summation because it is the same for all pipes in the circuit and absolute-value signs have been added to account for the direction of summation around the circuit. The last equation is solved for ΔQ in each circuit in the network

$$\Delta Q = -\frac{\Sigma rQ_0\,|Q_0|^{n-1}}{\Sigma rn\,|Q_0|^{n-1}} \tag{10.7.2}$$

When ΔQ is applied to each pipe in a circuit in accordance with Eq. (10.7.1), the directional sense is important; i.e., it adds to flows in the clockwise direction and subtracts from flows in the counterclockwise direction.

Steps in an arithmetic procedure may be itemized as follows:

1. Assume the best distribution of flows that satisfies continuity by careful examination of the network.
2. For each pipe in an elementary circuit, calculate and sum the net head loss $\Sigma h_f = \Sigma rQ^n$. Also calculate $\Sigma rn\,|Q|^{n-1}$ for the circuit. The negative ratio, by Eq. (10.7.2) yields the correction, which is then added algebraically to each flow in the circuit to correct it.
3. Proceed to another elementary circuit and repeat the correction process of 2. Continue for all elementary circuits.
4. Repeat 2 and 3 as many times as needed until the corrections (ΔQ's) are arbitrarily small.

The values of r occur in both numerator and denominator; hence, values proportional to the actual r may be used to find the distribution. Similarly, the apportionment of flows may be expressed as a percent of the actual flows. To find a particular head loss, the actual values of r and Q must be used after the distribution has been determined.

Example 10.8 The distribution of flow through the network of Fig. 10.13 is desired for the inflows and outflows as given. For simplicity n has been given the value 2.0.
The assumed distribution is shown in diagram *a*. At the upper left the term $\Sigma rQ_0\,|Q_0|^{n-1}$ is computed for the lower circuit numb... ...ext to the diagram ... the left ... compt... of

$70^2 \times 6 = 29\,400$	$2 \times 70 \times 6 = 840$	$15^2 \times 1 = 225$	$2 \times 15 \times 1 = 30$
$35^2 \times 3 = 3\,675$	$2 \times 35 \times 3 = 210$	$-35^2 \times 2 = -2\,450$	$2 \times 35 \times 2 = 140$
$-30^2 \times 5 = -4\,500$	$2 \times 30 \times 5 = 300$	$-13.83^2 \times 3 = -574$	$2 \times 13.83 \times 3 = 83$
$28\,575$	$1\,350$	$-2\,799$	253

$$\Delta Q_1 = -\frac{28\,575}{1\,350} = -21.17 \qquad \Delta Q_2 = \frac{2\,799}{253} = 11.06$$

$48.83^2 \times 6 = 14\,308$	$2 \times 48.83 \times 6 = 586$	$26.06^2 \times 1 = 679$	$2 \times 26.06 \times 1 = 52$
$2.77^2 \times 3 = 23$	$2 \times 2.77 \times 3 = 17$	$-23.94^2 \times 2 = -1\,146$	$2 \times 23.94 \times 2 = 96$
$-51.17^2 \times 5 = -13\,090$	$2 \times 51.17 \times 5 = 511$	$-1.656^2 \times 3 = -8$	$2 \times 1.656 \times 3 = 10$
$1\,241$	$1\,114$	-475	158

$$\Delta Q_1 = -\frac{1\,241}{1\,114} = -1.114 \qquad \Delta Q_2 = \frac{475}{158} = 3.006$$

$\Delta Q_1 = 0.0079$ $\quad \Delta Q_2 = 0.169$

$\Delta Q_1 = 0.0013$ $\quad \Delta Q_2 = 0.0003$

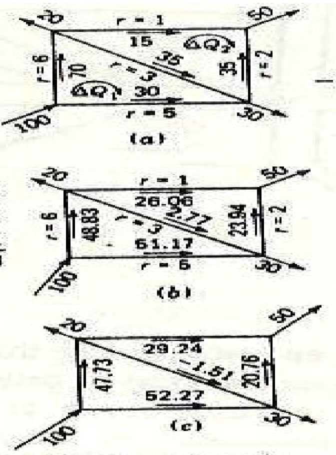


Figure 10.13 Solution for flow in a simple network.

$\Sigma nr\,|Q_0|^{n-1}$ for the same circuit. The same format is used for the second circuit in the upper righ of the figure. The corrected flow after the first step for the top horizontal pipe is determined a $15 + 11.06 = 26.06$ and for the diagonal as $35 + (-21.17) + (-11.06) = 2.77$. Diagram (*b*) show the flows after one correction. Diagram (*c*) shows tne values after four corrections.

Very simple networks, such as the one shown in Fig. 10.13, may be solved with the hand-held programmable calculator if it has memory storage of about 15 and about 100 program steps. For networks larger than the previous example or for networks that contain multiple reservoirs, supply pumps, or booster pumps, the Hardy Cross loop-balancing method may be programmed for numerical solution on a digital computer. Such a program is provided in the next section.

A number of more general methods†,‡,§ are available, primarily based upon the Hardy Cross loop-balancing or node-balancing schemes. In the more general methods the system is normally modeled with a set of simultaneous equations which are solved by the Newton-Raphson method. Some programmed solutions‡,§ are very useful as design tools, since pipe sizes or roughnesses may be treated as unknowns in addition to junction pressures and flows.

† R. Epp and A. G. Fowler, Efficient Code for Steady-State Flows in Networks, *J. Hydraul. Div. ASCE*, vol. 96, no. HY1, pp. 43–56, January 1970.

‡ Uri Shamir and C. D. D. Howard, Water Distribution Systems Analysis, *J. Hydraul. Div., ASCE*, vol. 94, no. HY1, pp. 219–234, January 1968.

§ Michael A. Stoner, A New Way to Design Natural Gas Systems, *Pipe Line Ind.*, vol. 32, no. 2

10.8 COMPUTER PROGRAM FOR STEADY-STATE HYDRAULIC SYSTEMS

Hydraulic systems that contain components different from pipelines can be handled by replacing the component with an equivalent length of pipeline. When the additional component is a pump, special consideration is needed. Also, in systems that contain more than one fixed hydraulic-grade-line elevation, a special artifice must be introduced.

For systems with multiple fixed-pressure-head elevations, Fig. 10.14, *pseudo loops* are created to account for the unknown outflows and inflows at the reservoirs and to satisfy continuity conditions during balancing. A pseudo loop is created by using an imaginary pipeline that interconnects each pair of fixed pressure levels. These imaginary pipelines carry no flow but maintain a fixed drop in the hydraulic grade line equal to the difference in elevation of the reservoirs. If head drop is considered positive in an assumed positive direction in the imaginary pipe, then the correction in loop 3, Fig. 10.14, is

$$\Delta Q_3 = -\frac{150 - 135 - r_4 Q_4 |Q_4|^{n-1} - r_1 Q_1 |Q_1|^{n-1}}{n r_4 |Q_4|^{n-1} + n r_1 |Q_1|^{n-1}} \tag{10.8.1}$$

This correction is applied to pipes 1 and 4 only. If additional real pipelines existed in a pseudo loop, each would be adjusted accordingly during each loop-balancing iteration. The terms in Eq. (10.8.1) may be identified easily by relating to Eq. (10.7.2). Alternatively, the same equation may be generated by application of Newton's method (Appendix B).

A pump in a system may be considered as a flow element with a negative head loss equal to the head rise that corresponds to the flow through the unit. The pump-head-discharge curve, element 8 in Fig. 10.14, may be expressed by a cubic equation

$$H = A_0 + A_1 Q_8 + A_2 Q_8^2 + A_3 Q_8^3$$

where A_0 is the shutoff head of the pump. The correction in loop 4 is

$$\Delta Q_4 = -\frac{135 - 117 - (A_0 + A_1 Q_8 + A_2 Q_8^2 + A_3 Q_8^3) + r_5 Q_5 |Q_5|^{n-1}}{n r_5 |Q_5|^{n-1} - (A_1 + 2A_2 Q_8 + 3A_3 Q_8^2)} \tag{10.8.2}$$

This correction is applied to pipe 5 and to pump 8 in the loop. Equation (10.8.2) is developed by application of Newton's method to the loop. For satisfactory balancing of networks with pumping stations, the slope of the head-discharge curve should always be less than or equal to zero.

The FORTRAN IV program (Fig. 10.15) may be used to analyze a wide

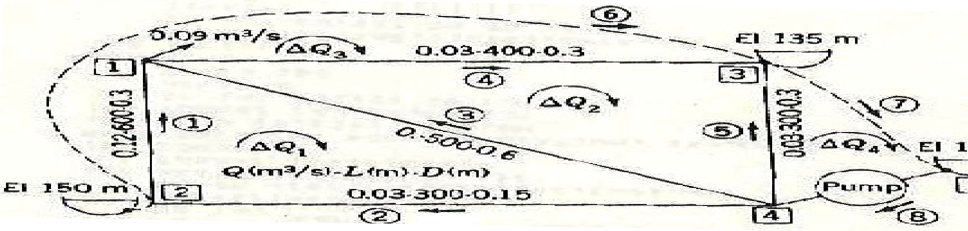


Figure 10.14 Sample network.

variety of liquid steady-state pipe flow problems. The Hardy Cross loop-balancing method is used. Pipeline flows described by the Hazen-Williams equation or laminar or turbulent flows analyzed with the Darcy-Weisbach equation can be handled; multiple reservoirs or fixed pressure levels, as in a sprinkler system, can be analyzed; and systems with booster pumps or supply pumps can be treated.

A network is visualized as a combination of elements that are interconnected at junctions. The elements may include pipelines, pumps, and imaginary elements which are used to create pseudo loops in multiple-reservoir systems. All minor losses are handled by estimating equivalent lengths and adding them onto the actual pipe lengths. Each element in the system is numbered up to a maximum of 100, without duplication and not necessarily consecutively. A positive flow direction is assigned to each element, and, as in the arithmetic solution, an estimated flow is assigned to each element such that continuity is satisfied at each junction. The assigned positive flow direction in a pump must be in the intended direction of normal pump operation. Any solution with backward flow through a pump is invalid. The flow direction in the imaginary element that creates a pseudo loop indicates only the direction of fixed positive head drop, since the flow must be zero in this element. Each junction, which may represent the termination of a single element or the intersection of many elements, is numbered up to a maximum of 100, without duplication and not necessarily sequentially. An outflow or inflow at a junction is defined during the assignment of initial element flows.

The operation of the program is best visualized in two major parts: the first performs the balancing of each loop in the system successively and then repeats in an iterative manner until the sum of all loop flow corrections is less than a specified tolerance. At the end of this balancing process the element flows are computed and printed. The second part of an analysis involves the computation of the hydraulic-grade-line elevations at junctions in the system. Each of these parts requires a special indexing of the system configuration in the input data. Th

```
C HARDY CROSS LOOP BALANCING INCLUDING MULTIPLE RESERVOIRS & PUMPS(PU)
C HAZEN-WILLIAMS(HW) OR DARCY-WEISBACH(DW) MAY BE USED FOR PIPES
C ENGLISH(EN) OR SI UNITS(SI) MAY BE USED. POSITIVE DH IN ELEMENT IS HEAD
  DROP DIMENSION ITY(4),ID(2),ITYPE(100),ELEM(500),IND(500),Q(100),H(100)
     2,S(20),IX(240)
      DATA ITY/'HW','DW','PS','PU'/,IE/'&&'/,ID/'EN','SI'/
   10 DO 12 J=1,500
      IF (J.LE.100)ITYPE(J)=5
      IF (J.LE.100) H(J)=-1000.
      IF (J.LE.240)IX(J)=0
   12 IND(J)=0
C READ PARAMETERS FOR PROBLEM, AND ELEMENT DATA
      READ (5,15,END=99) NT,KK,TOL,VNU,DEF
   15 FORMAT (A2,I8,F10.4,F10.7,F10.5)
      IF (NT.EQ.ID(2))GO TO 20
      WRITE (6,18) VNU
   18 FORMAT (' ENGLISH UNITS SPECIFIED, VISCOSITY IN FT**2/SEC=',F10.7)
      UNITS=4.727
      G=32.174
      GO TO 22
   20 WRITE (6,21) VNU
   21 FORMAT (' SI UNITS SPECIFIED, VISCOSITY IN M**2/SEC=',F10.7)
      UNITS=10.674
      G=9.806
   22 WRITE (6,24) TOL,KK
   24 FORMAT (' DESIRED FLOW TOLERANCE=',F5.3,' NO. OF ITERATIONS=',I5//
     2' PIPE       Q(CFS OR M**3/S)   L(FT OR M)   D(FT OR M)   HW C OR EPS')
   26 READ (5,30) NT,I,QQ,X1,X2,X3,X4,X5
   30 FORMAT (A2,3X,I5,3F10.3,F10.5,2F10.3)
      IF (NT.EQ.IE) GO TO 68
      Q(I)=QQ
      DO 32 NTY=1,4
      IF (NT.EQ.ITY(NTY))GO TO 33
   32 CONTINUE
   33 ITYPE(I)=NTY
      KP=4*(I-1)+1
      GO TO (41,42,53,64),NTY
   41 IF (X3.EQ.0.)X3=DEF
      ELEM(KP)=UNITS*X1/(X3**1.852*X2**4.8704)
      EX=1.852
      GO TO 43
   42 IF (X3.EQ.0.)X3=DEF
      EX=2.
      ELEM(KP)=X1/(2.*G*X2**5*.7854*.7854)
      ELEM(KP+1)=1./(.7854*X2*VNU)
      ELEM(KP+2)=X3/(3.7*X2)
   43 WRITE (6,45)I,Q(I),X1,X2,X3
   45 FORMAT (I5,F18.3,F12.1,F12.3,F14.5)
      EN=EX-1.
      GO TO 26
   53 ELEM(KP)=X1
      WRITE (6,55) I,X1
   55 FORMAT (I5,' RESERVOIR ELEV DIFFERENCE=',F10.2)
      GO TO 26
   64 ELEM(KP)=X2
      ELEM(KP+3)=(X5-3.*(X4-X3)-X2)/(6.*X1**3)
      ELEM(KP+2)=(X4-2.*X3+X2)/(2.*X1**2)-ELEM(KP+3)*3.*X1
      ELEM(KP+1)=(X3-X2)/X1-ELEM(KP+2)*X1-ELEM(KP+3)*X1*X1
      WRITE (6,66) I,X1,X2,X3,X4,X5,(ELEM(KP+J-1),J=1,4)
   66 FORMAT (I5,' PUMP CURVE, DQ=',F7.3,' H=',4F8.1/5X,
     2' COEF IN PUMP EQ=',4F11.3)
      GO TO 26
C READ LOOP INDEXING DATA, IND=NO. PIPES,PIPE,PIPE,ETC. CLOCKWISE+,CC-
   68 I1=1
   70 I2=I1+14
      READ (5,75) NT,(IND(I),I=I1,I2)
   75 FORMAT (A2,2X,15I4)
      IF (NT.EQ.IE) GO TO 78
      I1=I2+1
      GO TO 70
   78 IF (I1.EQ.1) GO TO 140
      WRITE (6,79) (IND(I),I=1,I1)
   79 FORMAT (' IND='/(15I4))
```

```
C BALANCE ALL LOOPS
      DO 130 K=1,KK
      DDQ=0.
      IP=1
80    I1=IND(IP)
      IF(I1.LE.0) GO TO 124
      DH=0.
      HDQ=0.
      DO 110 J=1,I1
      I=IND(IP+J)
      IF (I) 81,110,82
81    S(J)=-1.
      I=-I
      GO TO 83
82    S(J)=1.
83    NTY=ITYPE(I)
      KP=4*(I-1)+1
      GO TO (91,92,103,104),NTY
91    R=ELEM(KP)
      GO TO 95
92    REY=ELEM(KP+1)*ABS(Q(I))
      IF(REY.LT.1.) REY=1.
      IF (REY-2000.) 93,94,94
93    R=ELEM(KP)*64./REY
      GO TO 95
94    R=ELEM(KP)*1.325/(ALOG(ELEM(KP+2)+5.74/REY**.9))**2
95    DH=DH+S(J)*R*Q(I)*ABS(Q(I))**EN
      HDQ=HDQ+EX*R*ABS(Q(I))**EN
      GO TO 110
103   DH=DH+S(J)*ELEM(KP)
      GO TO 110
104   DH=DH-S(J)*(ELEM(KP)+Q(I)*(ELEM(KP+1)+Q(I)*(ELEM(KP+2)+Q(I)*
     2ELEM(KP+3))))
      HDQ=HDQ-(ELEM(KP+1)+2.*ELEM(KP+2)*Q(I)+3.*ELEM(KP+3)*Q(I)**2)
110   CONTINUE
      IF (ABS(HDQ).LT..0001) HDQ=1.
      DQ=-DH/HDQ
      DDQ=DDQ+ABS(DQ)
      DO 120 J=1,I1
      I=IABS(IND(IP+J))
      IF (ITYPE(I).EQ.3) GO TO 120
      Q(I)=Q(I)+S(J)*DQ
120   CONTINUE
      IP=IP+I1+1
      GO TO 80
124   WRITE (6,125) K,DDQ
125   FORMAT (' ITERATION NO.',I4,' SUM OF FLOW CORRECTIONS=',F10.4)
      IF (DDQ.LT.TOL) GO TO 140
130   CONTINUE
140   WRITE (6,141)
141   FORMAT (' ELEMENT    FLOW')
      DO 150 I=1,100
      NTY=ITYPE(I)
      GO TO (142,142,150,142,150),NTY
142   WRITE (6,143) I,Q(I)
143   FORMAT (I5,F10.3)
150   CONTINUE
C READ DATA FOR HGL COMPUTATION; IX=JUNC,ELEMENT,JUNC,ELEM,JUNC,ETC.
152   READ (5,155) NT,K,HH
155   FORMAT (A2,I8,F10.3)
      IF (NT.EQ.IE) GO TO 160
      H(K)=HH
      GO TO 152
160   I1=1
162   I2=I1+14
      READ (5,75)NT,(IX(K),K=I1,I2)
      IF (NT.EQ.IE) GO TO 170
      I1=I2+1
      GO TO 162
170   WRITE (6,171) (IX(I),I=1,I1)
171   FORMAT ('   IX='/(15I4))
      IP=1
180   DO 200 J=1,238,2
      IF (J.EQ.1) I1=IX(IP)
      I=IX(IP+J)
      N=IX(IP+J+1)
      IF(I) 181,199,182
181   SS=-1.
      I=-I
      GO TO 183
182   SS=1.
183   NTY=ITYPE(I)
      KP=4*(I-1)+1
      GO TO (184,185,189,190,199),NTY
184   R=ELEM(KP)
      GO TO 188
185   REY=ELEM(KP+1)*ABS(Q(I))
      IF (REY.LT.1.) REY=1.
      IF (REY-2000.) 186,187,187
186   R=ELEM(KP)*64./REY
      GO TO 188
187   R=ELEM(KP)*1.325/(ALOG(ELEM(KP+2)+5.74/REY**.9))**2
188   H(N)=H(I1)-SS*R*Q(I)*ABS(Q(I))**EN
      GO TO 199
189   H(N)=H(I1)-SS*ELEM(KP)
      GO TO 199
190   H(N)=H(I1)+SS*(ELEM(KP)+Q(I)*(ELEM(KP+1)+Q(I)*(ELEM(KP+2)+Q(I)*
     2ELEM(KP+3))))
199   IF (IX(J+IP+3).EQ.0) GO TO 210
      IF (IX(J+IP+2).EQ.0) GO TO 205
200   I1=N
205   IP=IP+J+3
      GO TO 180
210   WRITE (6,215)
215   FORMAT (' JUNCTION   HEAD')
      DO 220 N=1,100
      IF (H(N).EQ.-1000.) GO TO 220
      WRITE (6,143) N,H(N)
220   CONTINUE
      GO TO 10
99    STOP
      END
```

Figure 10.15 FORTRAN program for hydraulic systems.

indexing of the system loops for balancing is placed in the vector IND. A series of integer values identifies each loop sequentially by the number of elements in the loop followed by the element number of each element in the loop. The directional sense of flow in each element is identified by a positive element number for the clockwise direction and a negative element number for counterclockwise. The second part of the program requires an identification of one or more junctions with known heads. Then a series of junction and element numbers indexes a continuous path through the system to all junctions where the hydraulic grade line is wanted. The path may be broken at any point by an integer zero followed by a new junction where the head is known. These data are stored in the vector IX by a junction number where the head is known followed by a contiguous element number and junction number. Again the positive element number is used in the assigned flow direction, and the negative element number is used when tracing a path against the assigned element flow direction. Any continuous path may be broken by inserting a zero; then a new path is begun with a new initial junction, an element, and a node, etc. All junction hydraulic-grade-line elevations that are computed are printed.

As shown below, the type of each element is identified in the input data, and each element is identified in the program by the assignment of a unique numerical value in the vector ITYPE.

Element	Data	Program	Element	Data	Program
Hazen-Williams pipeline	HW	1	Pseudo element	PS	3
Darcy-Weisbach pipeline	DW	2	Pump	PU	4

The physical data associated with each element are entered on separate cards. In the program the physical data that describe all elements in the system are stored in the vector ELEM, with five locations reserved for each element. As an example of the position of storage of element information, the data pertaining to element number 13 are located in positions 61 to 65 in ELEM.

Data preparation for the program is best visualized in four steps, as shown in Fig. 10.16 and described below. Formated input is used, as shown in Fig. 10.16 and the program.

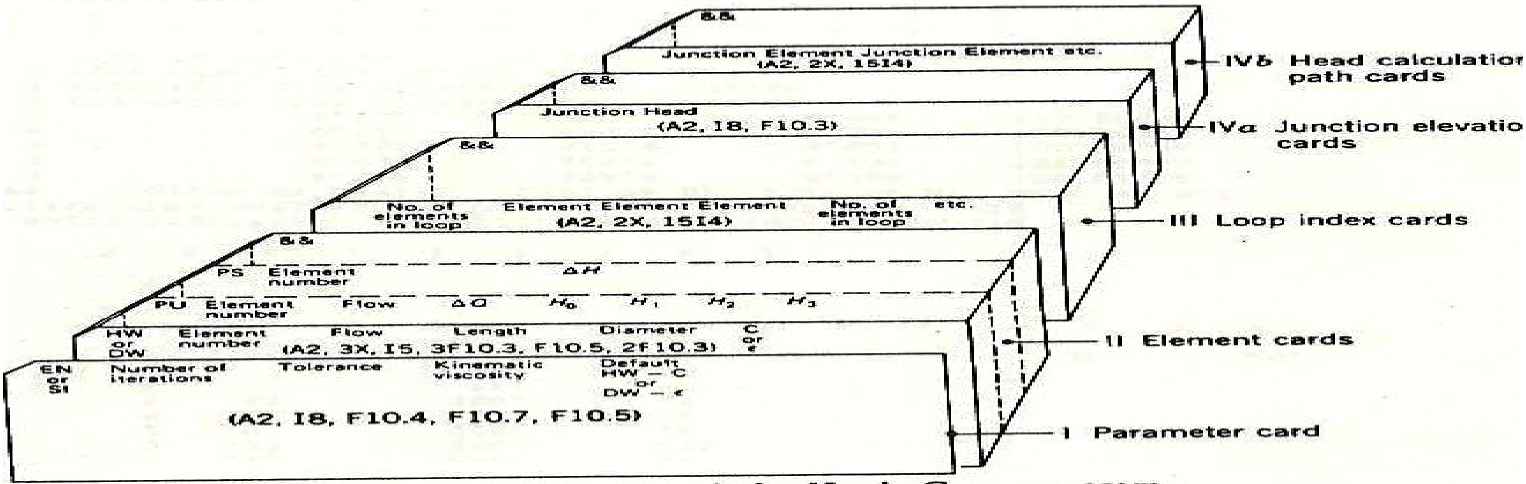


Figure 10.16 Data cards for Hardy Cross program.

Step 1: Parameter Description Card

The type of unit to be used in the analysis is defined by the characters SI for the SI units. An integer defines the maximum number of iterations to be allowed during the balancing scheme. An acceptable tolerance is set for the sum of the absolute values of the corrections in each loop during each iteration. The liquid kinematic viscosity must be specified if the Darcy-Weisbach equation is used for pipeline losses. If the Hazen-Williams equation is used, a default value for the coefficient C may be defined, or if the Darcy-Weisbach equation is used, a default value for absolute pipe roughness may be defined. If the default value is used on the parameter card, it need not be placed on the element cards; however, if it is, the element data override the default value.

Step 2: Element Cards

Each element in the system requires a separate card. Pipeline elements require either HW or DW to indicate the equation for the problem solution, the element number, the estimated flow, the length, the inside diameter, and (if the default value is not used) either the Hazen-Williams coefficient or the pipe roughness for the Darcy-Weisbach equation. Pump elements require PU to indicate the element type, the element number, the estimated flow, a flow increment ΔQ at which values of pump head are specified, and four values of head from the pump-characteristic curve beginning at shutoff head and at equal flow intervals of ΔQ. The pseudo element for the pseudo loop requires PS to indicate the type, the element number, a zero or blank for the flow, and a difference in elevation between the interconnected fixed-pressure-head levels with head drop positive. The end of the element data is indicated by a card with "&&" in the first two columns.

Step 3: Loop Index Cards

These data are supplied with 15 integer numbers per card in the following order: the number of elements in a loop (maximum of 20) followed by the element number of each element in the loop with a negative sign to indicate counterclockwise flow direction. This information is repeated until all loops are defined. The end of step 3 data is indicated by a card with "&&" in columns 1 and 2.

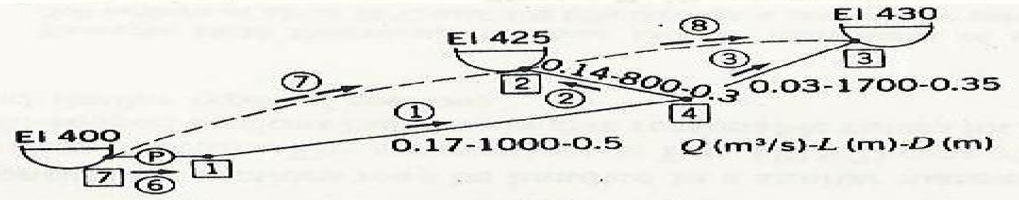
Step 4: Head Calculation Cards

Junctions with fixed elevations are identified on separate cards by giving the junction number and the hydraulic-grade-line elevation. There must be one or more of these cards followed by a card with "&&" in the first two columns to indicate the end of this type of data.

The path to be followed in computing the hydraulic-grade-line elevations is specified by supplying 15 integer values per card in the following order: junction

```
SI        30     .001          .000001       100.
HW         1     .12            600.          .3
HW         2     .03            300.          .15
HW         3     .0             500.          .6
HW         4     .03            400.          .3
HW         5     .03            300.          .3
PS         6                     15.
PS         7                     18.
PU         8     .06            .03          30.          29.          26.          20
&&
      3    2    1   -3    3    4   -5    3    3    6   -4   -1    3    5    7
      8
&&
           5    117.
&&
      5    8    4    2    2    1    1    4    3
&&
SI UNITS SPECIFIED, VISCOSITY IN M**2/SEC= 0.0000010
DESIRED FLOW TOLERANCE=0.001 NO. OF ITERATIONS=   30
PIPE       Q(CFS OR M**3/S)    L(FT OR M)    D(FT OR M)    HW C OR EPS
   1             0.120            600.0         0.300       100.00000
   2             0.030            300.0         0.150       100.00000
   3             0.000            500.0         0.600       100.00000
   4             0.030            400.0         0.300       100.00000
   5             0.030            300.0         0.300       100.00000
   6  RESERVOIR  ELEV  DIFFERENCE=       15.00
   7  RESERVOIR  ELEV  DIFFERENCE=       18.00
   8  PUMP CURVE, DQ=   0.030 H=       30.0        29.0        26.0        20.0
      COEF IN PUMP EQ=       30.000      -11.111     -555.556    -6172.836
IND=
  3    2    1   -3    3    4   -5    3    3    6   -4   -1    3    5    7
  8    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  0
ITERATION NO.    1 SUM OF FLOW CORRECTIONS=      0.1385
ITERATION NO.    2 SUM OF FLOW CORRECTIONS=      0.1040
ITERATION NO.    3 SUM OF FLOW CORRECTIONS=      0.0372
ITERATION NO.    4 SUM OF FLOW CORRECTIONS=      0.0034
ITERATION NO.    5 SUM OF FLOW CORRECTIONS=      0.0006
ELEMENT      FLOW
   1         0.143
   2        -0.034
   3         0.027
   4         0.080
   5         0.094
   8         0.087
 IX=
  5    8    4    2    2    1    1    4    3    0    0    0    0    0    0
  0
JUNCTION    HEAD
   1      137.811
   2      150.044
   3      135.044
   4      137.797
   5      117.000
```

Figure 10.17 Program input and output for Example 10.9.




```
SI        30     .001          .000001        120.
HW         1     0.17           1000.           0.5
HW         2     0.14            800.           0.3
HW         3     0.03           1700.           0.35
PU         6     0.17              0.2         36.          34.          30.          24.
PS         7     0.             -25.
PS         8     0.              -5.
&&
     4    7   -2   -1   -6    3    8   -3    2
&&
          7    400.
&&
     7    6    1    1    4    2    2    0    4    3    3
&&
```

Figure 10.18 Input data for branching-pipe system in U.S. customary units with Hazen-Williams formula.

number where the head is known, element number (with a negative sign to indicate a path opposite to the assumed flow direction), junction number, etc. If one wants a new path to begin at a junction different from the last listed junction, a single zero is added, followed by a junction where the head is known, element number, junction number, etc. The end of step 4 data is indicated by a card with "&&" in columns 1 and 2.

Example 10.9 The program in Fig. 10.15 is used to solve the network problem displayed in Fig. 10.14. The pump data are as follows:

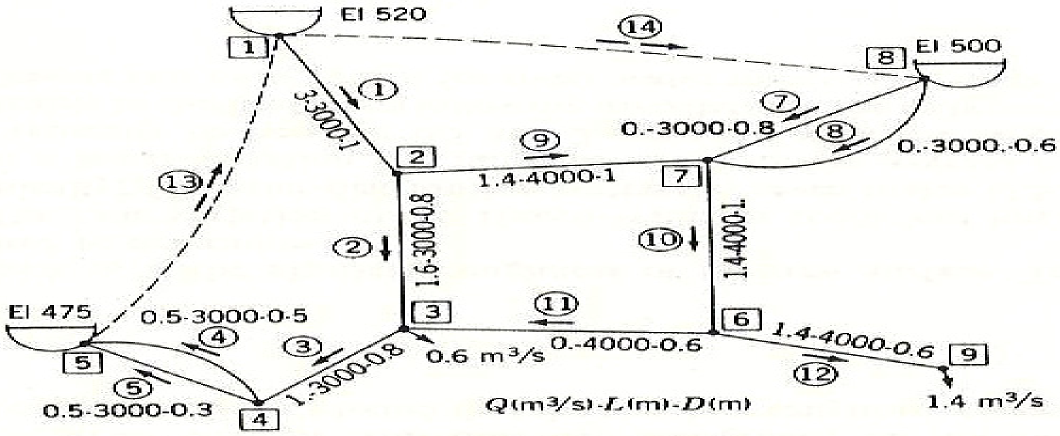
Q, m³/s	0	0.03	0.06	0.09
H, m	30	29	26	20

The Hazen-Williams pipeline coefficient for all pipes is 100. Figure 10.17 displays the input data and the computer output for this problem.

Figures 10.18 to 10.20 give input data for three systems which can be solved with this program.

10.9 CONDUITS WITH NONCIRCULAR CROSS SECTIONS

In this chapter so far, only circular pipes have been considered. For cross sections that are noncircular, the Darcy-Weisbach equation may be applied if the term D can be interpreted in terms of the section. The concept of the *hydraulic radius R*




```
SI        30      .002       .0000012      .0005
DW         1      3.         3000.         1.
DW         2      1.6        3000.         0.8
DW         3      1.0        3000.         0.8
DW         4       .5        3000.         0.5
DW         5       .5        3000.         0.3
DW         7      0.         3000.         0.8
DW         8      0.         3000.         0.6
DW         9      1.4        4000.         1.0
DW        10      1.4        4000.         1.
DW        11      0.         4000.         0.6
DW        12      1.4        4000.         0.6
PS        13      0.         -45.
PS        14      0.          20.
&&
      5    1    2    3    4   13    4    9   10   11   -2    2   -7    8    2
     -4    5    4   14    7   -9   -1
           1       520.
&&
      1    1    2    2    3    3    4    4    5    0    3  -11    6   12    9
&&    0    2    9    7   -8    8
```

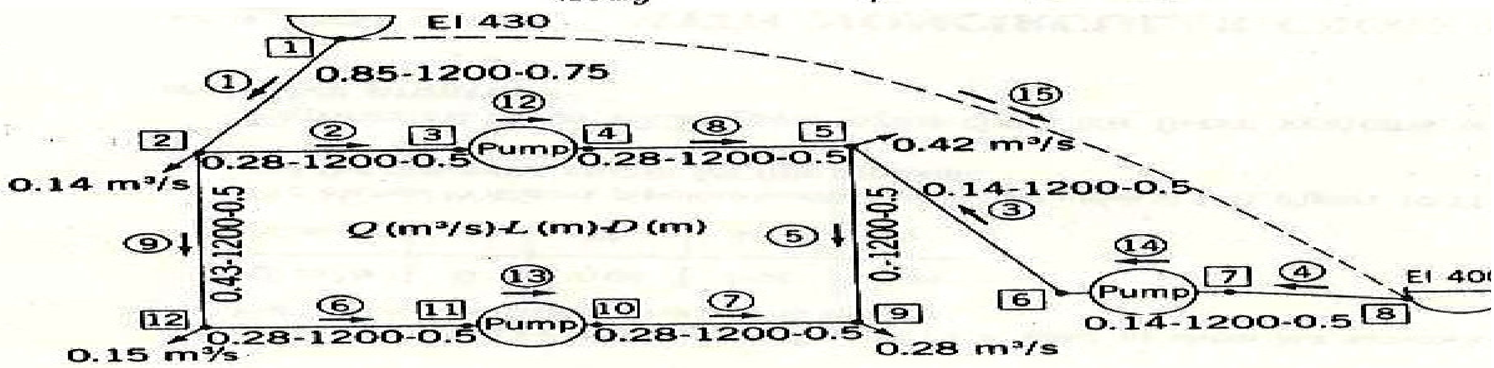
Figure 10.19 Input data for hydraulic system. SI units and Darcy-Weisbach equation.

permits circular and noncircular sections to be treated in the same manner. The hydraulic radius is defined as the cross-sectional area divided by the *wetted perimeter*. Hence, for a circular section,

$$R = \frac{\text{area}}{\text{perimeter}} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} \qquad (10.9.1)$$

and the diameter is equivalent to $4R$. Assuming that the diameter may be replaced by $4R$ in the Darcy-Weisbach equation, in the Reynolds number, and in the relative roughness,

$$h_f = f\,\frac{L}{4R}\,\frac{V^2}{2g} \qquad \mathbf{R} = \frac{V4R\rho}{\mu} \qquad \frac{\epsilon}{D} = \frac{\epsilon}{4R} \qquad (10.9.2)$$




```
SI        30      .001       .000001       120.
HW         1      0.85       1200.         0.75
HW         2      0.28       1200.         0.5
HW         3      0.14       1200.         0.5
HW         4      0.14       1200.         0.5
HW         5      0.         1200.         0.5
HW         6      0.28       1200.         0.5
HW         7      0.28       1200.         0.5
HW         8      0.28       1200.         0.5
HW         9      0.43       1200.         0.5
PU        12      0.28         30.         28.         24.         17.
PU        13      0.28         33.         31.         26.5        20.
PU        14      0.14         45.         43.         39.         32.
PS        15      0.           30.
&&
      8    2   12    8    5   -7  -13   -6   -9    8   15    4   14    3   -8
    -12   -2   -1
&&
           1       430.
           8       400.
&&
      1    1    2    2    3   12    4    8    5    5    9   -7   10  -13   11
     -6   12    0    8    4    7   14    6
&&
```

Figure 10.20 Input for booster-pump system.

Noncircular sections may be handled in a similar manner. The Moody diagram applies as before. The assumptions in Eqs. (10.9.2) cannot be expected to hold for odd-shaped sections but should give reasonable values for square, oval, triangular, and similar types of sections.

Example 10.10 Determine the head loss, in millimetres of water, required for flow of 300 m^3/min of air at 20°C and 100 kPa through a rectangular galvanized-iron section 700 mm wide, 350 mm high, and 70 m long.

$$R = \frac{A}{P} = \frac{0.7 \times 0.35}{2(0.7 + 0.35)} = 0.117 \text{ m} \qquad \frac{\epsilon}{4R} = \frac{0.00015}{4 \times 0.117} = 0.00032$$

$$V = \frac{300}{60(0.7)(0.35)} = 20.41 \text{ m/s} \quad \mu = 2.2 \times 10^{-5} \text{ Pa}\cdot\text{s} \quad \rho = \frac{p}{R'T} = \frac{100\,000}{287(273 + 20)} = 1.189 \text{ kg/m}^3$$

$$\mathbf{R} = \frac{VD\rho}{\mu} = \frac{V4R\rho}{\mu} = \frac{20.41 \times 4 \times 0.117 \times 1.189}{2.2 \times 10^{-5}} = 516\,200$$

From Fig. 5.32, $f = 0.0165$ $\quad h_f = f\,\frac{L}{4R}\,\frac{V^2}{2g} = 0.0165\,\frac{70}{4 \times 0.117}\,\frac{20.41^2}{2 \times 9.806} = 52.42 \text{ m}$

The unit gravity force of air is $\rho g = 1.189 \times 9.806 = 11.66 \text{ N/m}^3$. In millimetres of water,

$$\frac{52.42 \times 11.66 \times 1000}{9806} = 62.33 \text{ mm}$$

10.10 AGING OF PIPES

The Moody diagram, with the values of absolute roughness shown there, is for new, clean pipe. With use, pipes become rougher, owing to corrosion, incrustations, and deposition of material on the pipe walls. The speed with which the friction factor changes with time depends greatly on the fluid being handled. Colebrook and White† found that the absolute roughness ϵ increases linearly with time,

$$\epsilon = \epsilon_0 + \alpha t \tag{10.10.1}$$

in which ϵ_0 is the absolute roughness of the new surface. Tests on a pipe are required to determine α.

The time variation of the Hazen-Williams coefficient has been summarized graphically‡ for water-distribution systems in seven major U.S. cities. Although it is not a linear variation, the range of values for the average rate of decline in C may typically be between 0.5 and 2 per year, with the larger values generally applicable in the first years following installation. The only sure way that accurate coefficients can be obtained for older water mains is through field tests.

† C. F. Colebrook and C. M. White, The Reduction of Carrying Capacity of Pipes with Age, *J. Inst. Civ. Eng. Lond.*, 1937.

‡ W. D. Hudson, Computerized Pipeline Design, *Transp. Eng. J. ASCE*, vol. 99, no. TE1, 1973.

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